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Under-relaxed Iterative Procedure for Feasible Short-Term Scheduling of a Hydro Chain

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***Abstract* — This paper presents a method for short-term hydro scheduling. The objective is to determine a feasible operation of a set of coupled hydro units, fulfilling the aggregate requirements obtained from a higher-level hydrothermal coordination tool. In order to take into account the nonlinear relationship among the electrical power, the net head and the turbine water discharge, an under-relaxed iterative procedure is proposed. The performance of this algorithm enhances previous research works as possible diverging oscillations are damped in order to reach the convergence. Therefore, the net heads used to build the power/discharge curves of the head dependent units are determined using previous iterations results. This way, each stage can be solved by means of a MILP optimization problem where binary variables allow modeling the discrete hydro unit-commitment decisions. The process finishes when the maximum gap between the reservoir levels of two consecutive iterations satisfies the convergence tolerance. A case study is also presented to show its application to a real size hydro chain.**

***Index Terms* — hydroelectric power generation, mixed integer linear programming, short-term hydro scheduling, water rights.**

I. NOMENCLATURE

K, k	Set and index of periods
I, i	Set and index of hydro units
Ω_i	Subset of units direct upstream of unit *i*
ℓ_k	Duration of period *k* [h]
pd	Penalty factor for demand deviations [€·h/Hm^3]
ps	Penalty factor for spillage [€·h/Hm^3]
d_k	Hydro chain demand in period *k* [MW]
w_{ik}	Lateral inflows of unit *i* [Hm^3/h]
$\overline{\theta}_{ik}, \underline{\theta}_{ik}$	Water rights of unit *i* [Hm^3/h]
$\overline{p}_{ik}, \underline{p}_{ik}$	Output power limits of unit *i* in period *k* [MW]
$\overline{q}_{ik}, \underline{q}_{ik}$	Turbine discharge limits [Hm^3/h]
q_{ik}^{mxef}	Outflow at maximum efficiency point of unit *i* [MW]
p_{ik}^{mxef}	Output at maximum efficiency point of unit *i* [MW]
$\overline{v}_i, \underline{v}_i$	Reservoir capacity limits [Hm^3]
p_{ik}	Output power of unit *i* in k [MW]
h_{ik}	Net hydraulic head of unit *i* in k [MW]
v_{ik}	Reservoir level of unit *i* in k [Hm^3]
q_{ik}	Turbine discharge of unit *i* in *k* [Hm^3/h]
q_{ik}^{a}	Discharge over the minimum outflow [Hm^3/h]
q_{ik}^{b}	Discharge over maximum efficiency flow [Hm^3/h]
s_{ik}	Spillage of unit *i* in period *k* [Hm^3/h]
y_{ik}	Start up decision of unit *i* in *k* {0,1}
u_{ik}	Commitment of unit *i* in *k* {0,1}
z_{ik}	Shutdown decision of unit *i* in *k* {0,1}
c_i	Start up cost of unit *i* [€]
$\delta d_k^+ \ \delta d_k^-$	Demand deviations in *k* [MW]
α	Under-relaxation parameter

II. INTRODUCTION

The aim of this paper is to determine the feasible operation of the hydro generating units belonging to the same hydro chain, trying to meet an hourly power demand assigned previously, and accomplishing the reservoir level conditions at the beginning and at the end of the scheduling horizon. Other typical hydro constraints are also taken into account as for example water rights, detraction flows for water consumption, spillage management, etc. This problem could be part of a traditional hydrothermal coordination problem but also it could be applied as a sub-model of a profit-based scheduling master problem.

Hydro plants from the same chain can be connected in both series and in parallel, so the release of an upstream plant contributes to the inflow of downstream plants. To take advantage of its network structure, a natural approach to solve this scheduling problem is to model the system as a network flow model [1]. However, this approach fails when it is necessary to consider particular characteristics of the hydroelectric equipment that cannot be simplified: net hydraulic head effect, discrete unit-commitment decisions, forbidden operation areas in the non-linear input-output curves, etc. Other techniques that can be found in the related literature are dynamic programming [2], lagrangian relaxation [3, 4], linear programming [5], mixed integer linear programming (MILP) [6-9], optimal feedback control [10], artificial neural networks [11], etc.

One of the main difficulties related to the hydro scheduling problem is the nonlinear relationship among the hydroelectric generation, the water discharge flow, and the net hydraulic head of the corresponding reservoir [12]. Depending on the

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particular structure of the hydro sub-system, for short-term studies this dependence might be neglected or not. For instance, in a hydro sub-system formed by a few but very large reservoirs, a constant input-output curve for each one of them could be good enough as their variation during 24 hours would not represent a significant percentage of its original capacity. However, unlike thermal equipment, hydro plants have a wide variety of possible characteristics and configurations: from the mini-hydraulic installations (up to 5 MW) to the largest power plant on earth Itaipú (12600 MW).

In the Spanish system, there are several hydro chains formed by many but small reservoirs. In this case, even in the short-term, the head dependency has to be considered within the scheduling algorithm in order to obtain feasible and realistic results. Therefore, it is necessary to express accurately the hydropower plants characteristics and to consider the head variation effects on the output power. This paper is devoted to study and to solve the scheduling problem of this kind of hydro sub-systems.

Basically, the main strategies that can be found in the literature to cope with the head variation effect can be summarized as follows:

- To consider a constant average net head [8].
- To consider a discrete family of curves (up to 3 ó 5), predefined for the expected range of variation [9].
- To build an approximation of the truthful input-output surface by meshing and triangulation [7].
- To implement an iterative procedure where each iteration considers a fixed head, which is successively updated [5, 13].
- To make a very realistic and detailed modeling of the problem and apply computer simulation techniques, such as evolutionary programming, to find a near-optimum solution.

From the above items, this paper could be categorized in the fourth one. The major contributions of this paper with respect to previous and related work are the following ones:

1) We have experienced that applying the updating strategy proposed in [5, 13] could lead to oscillating solutions where convergence could not be reached at all. This happens especially for systems where the optimal reservoir management solution is very sensitive to the input-output curve considered and to the discrete commitment decisions of hydro units. For that reason, in this paper we propose to enhance the updating strategy by applying the under-relaxation technique used to search iteratively the solution of nonlinear equations in several variables [14].

2) Regarding [9], instead of deciding a priori the candidate input-output curves, the model proposed in this paper selects iteratively the most accurate ones. This way, the real output power computed with the water discharges corresponding to the last iteration solution coincide with the power variables used internally in the model. Therefore we avoid any kind of mismatch between the real output power that can be produced and the power variables within the decision process.

3) The method presented here overcomes the size limitations of [7], being suitable for larger hydro systems. Moreover, additional features are also included, such as considering the maximum efficiency points, including the start-up costs related to the loss of water, wear and tear o the equipment, etc. [15].

5) Regarding the optimization technique employed, the problem solved each iteration is stated as a MILP problem, written in GAMS and solved by CPLEX 7.1. This fact has two main advantages. The first one is the flexibility to introduce quite easily new features and capabilities in the model while not compromising the quality of the solution. The second one is the modularity. The hydro-scheduling problem can be embedded within another mixed integer linear programming problem (e.g. a traditional hydro-thermal unit-commitment, a pure hydro profit-based model for a price taker in a electricity market, etc.), where the iterations can be driven by the global MILP problem relaxing the head effect through the iterative process proposed.

This paper is organized as follows. First of all, the problem description and the model overview are described in section 3 where the iterative procedure is also explained in detail. Then, the mathematical formulation of the optimization problem solved each iteration is presented in section 4. After that, a real application of the proposed model is presented in section 5. Finally, concluding remarks are given in section 6.

III. MODEL OVERVIEW

A. Hydro Scheduling Problem (HSP) description

The objective of the hydro scheduling problem (HSP) is to find a feasible operation of a set of I hydro units during a temporal horizon divided in K periods, trying to meet a demand profile and accomplishing the reservoir level conditions.

In order to compute the global production of the hydro chain it is necessary to express accurately the hydroelectric generation functions for its hydro units. This can be attained introducing in the HSP the following equations:

$$p_{ik} = \Phi_i(q_{ik}, h_{ik}) \qquad \forall i, \forall k \qquad (1)$$

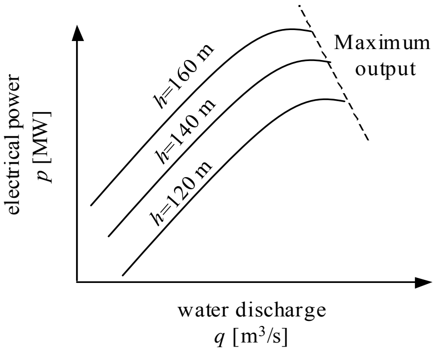


Figure 1: Input-output curves of a hydro unit with variable net head.

When h_{ik} is constant, only one curve is needed to characterize the generating unit. However, when the variation in the storage pond is a fairly large percentage of the overall net hydraulic head (h_{ik}), it cannot be considered constant. Moreover, the maximum capability of the plant will also tend to vary with the hydraulic head as shown in figure 1.Therefore, each hydro unit have to be characterized by its own function $\Phi_i(\cdot)$, which besides the efficiency curve of the turbine, the gravity constant and the water density, it might allow to model forbidden operating areas, multi-group effects, etc. Note that considering the multiplicity of input-output curves simultaneously could require a great computational effort and thus, many simplifications are usually found in the related literature.

B. Simplified Hydro Scheduling Problem (SHSP)

Consider the case where the net head values for each reservoir along the temporal horizon were known, h_{ik}^{υ}. In that case, we could build the following time-varying functions where the net head dependence has been removed by substituting in $\Phi_i(\cdot)$ those net head values:

$$\phi_{ik}^{\upsilon}(q_{ik}) = \Phi_i(q_{ik}, h_{ik}^{\upsilon}) \qquad \forall i, \forall k \qquad (2)$$

The hydro scheduling problem can be rewritten replacing the equations in (1) by the following ones,

$$p_{ik} = \phi_{ik}^{\upsilon}(q_{ik}) \qquad \forall i, \forall k \qquad (3)$$

This simplified problem will be denoted as SHSP (Simplified Hydro Scheduling Problem). It is important to note that the SHSP requires less computational effort to be solved than the HSP, as all the units are treated as non-head dependent. However, their solutions will remain different unless the functions $\phi_{ik}^{\upsilon}(\cdot)$ are the correct ones.

Let assume that the optimal solution of the HSP were known and denoted with an asterisk. In this case, the optimal power generation p_{ik}^{*} must satisfy:

$$p_{ik}^{*} = \Phi_i(q_{ik}^{*}, h_{ik}^{*}) \qquad \forall i, \forall k \qquad (4)$$

Thus, functions $\phi_{ik}^{\upsilon}(\cdot)$ leading to an equivalence between HSP and SHSP are the following ones:

$$\phi_{ik}^{*}(q_{ik}) = \Phi_i(q_{ik}, h_{ik}^{*}) \qquad \forall i, \forall k \qquad (5)$$

As the net heads are variables of the problem, univocally related to the reservoir levels at each time period, it is impossible to determine *a priori* the optimal evolution of net head values (h_{ik}^{*}). For that reason, the following under-relaxed iterative procedure is proposed.

C. Under-relaxed iIterative procedure

The proposed iterative procedure consists of the following steps:

Step 1) Iteration counter $\upsilon = 1$. Initialize h_{ik}^{υ}.

Step 2) Build $\phi_{ik}^{\upsilon}(q_{ik})$.

Step 3) Solve SHSP with $p_{ik} = \phi_{ik}^{\upsilon}(q_{ik}), \forall i, \forall k$.

Step 4) Update h_{ik}^{υ}. If convergence has not been reached, increase the iteration counter, ($\upsilon = \upsilon + 1$) and go to Step 2).

A detailed description of each step is presented hereafter.

Step 1) Assuming an average tailrace elevation, the net head, h [m], can be expressed simply as a function of the forebay elevation and, therefore, as a function of the reservoir storage, v [Hm^3].

$$h_i = \rho_i(v_i) \qquad (6)$$

In order to initialize h_{ik}^{υ}, we propose to apply (6), assuming a linear trajectory of the stored water by joining the initial and the final stored water, i.e. v_i^0 and v_i^f, along the temporal scope.

Step 2) Functions $\phi_{ik}^{\upsilon}(q_{ik})$ are built by applying (2) to the current values h_{ik}^{υ}.

Step 3) Once the hydro units have been characterized by their time-varying input-output functions, the SHSP solution can be obtained by solving the medium-scale MILP optimization problem presented in section IV.

In order to model the truthful input-output curve for a given net head, a piece-wise linear approximation is implemented. In the example shown in figure 2, two linear segments are used joining the minimum and maximum discharge points with the maximum efficiency power-outflow point. As the slope of the first segment is greater than the second one, no binary variables are needed to ensure that the interval $[\overline{q}, \underline{q}]$ is filled up correctly.

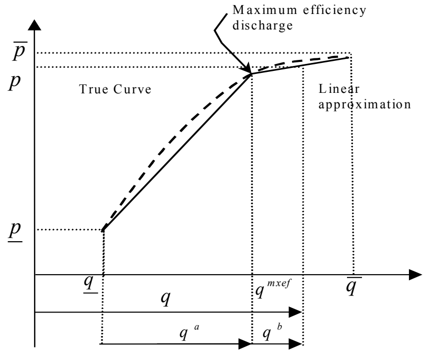


Figure 2: Input-Output curve approximation

However, it is necessary to introduce in the SHSP some binary variables in order to model the discrete unit-commitment, start-up and shutdown decisions. In this case, when the unit is off (i.e. u = 0), the water flow and the power produced must be zero. On the other hand, when the unit is on (i.e. u = 1), the unit has to discharge at least the minimum output flow $\underline{q}$, which is related to the minimum output power $\underline{p}$. Therefore, the total water discharge can be expressed as the unit-commitment decision multiplied by $\underline{q}$, plus the water discharge over the minimum output flow: $q = u \cdot \underline{q} + q^a + q^b$.

Step 4) The aim of this step is to check whether the convergence has been reached or not, and in this case, to prepare the input-data for the next iteration $\upsilon+1$.

The last solution of the SHSP provides new values for the reservoir levels $v_{ik}, \forall i \forall k$, and therefore by (6), these values could be used directly to update $h_{ik}^{\upsilon+1}$:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}\right) \qquad (7)$$

However, in order to avoid undesirable diverging oscillations, we propose to update the net heads using also previous iterations information. Let define the relaxation parameter $\alpha > 0$ [X1]. The updated net heads can be obtained by the following equation:

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}^{\upsilon+1}\right) = \rho_i\left(v_{ik}^{\upsilon} + \alpha \cdot \left[v_{ik} - v_{ik}^{\upsilon}\right]\right). \qquad (8)$$

Note that (7) is just a particular case of (8) when $\alpha = 1$. The selection of the best under-relaxation factor is empiric and unfortunately, it can be case-dependent. However, a general rule can be stated: for the early stages of iterations lower values of the under-relaxation factor will help to avoid divergence, and as the iterations get closer to the converged state, values very close to 1 help to speed up the progress.

Finally, it is necessary to check if convergence has been reached. This can be achieved defining the following error ε:

$$\varepsilon = \frac{(v_{jk} - v_{jk}^{\upsilon})}{v_{jk}^{\upsilon}} \qquad (9)$$

where j is the index of the reservoir in which the stored water in period k has the highest mismatch between two consecutive iterations. The iterative algorithm finishes when the value of ε is smaller than a given tolerance (e.g. 0.1 %).

IV. SHSP MATHEMATICAL FORMULATION

The hydro scheduling is formulated as a medium-scale MILP optimization problem. The following subsections include the mathematical formulation of objective function and the constraints involved in the problem.

A. Objective function

The main objective is to minimize the deviation between the total production of the hydro chain and the input demand that must be satisfied. The additional term includes the penalty factor for spillage and start-up costs.

$$\text{Minimize} \quad \sum_{k=1}^{K}\left(\delta d_k^{+} + \delta d_k^{-}\right) \cdot pd + \sum_{i=1}^{I}\sum_{k=1}^{K}\left(ps \cdot s_{ik} + c_i \cdot y_{ik}\right) \qquad (10)$$

B. Constraints

1) Reservoir water balance

For each reservoir at each time period, the following constraint establishes the water balance equation. The reservoir level at the beginning of period k+1 is the reservoir level at the beginning period k minus the released volume (discharged or spilled) plus the volume coming from direct upstream reservoirs and natural inflows.

$$v_{i(k+1)} = v_{ik} + \ell_k \cdot \left[-q_{ik} - s_{ik} + w_{ik} + \sum_{i \in \Omega_i}^{I}\left(q_{ik} + s_{ik}\right)\right] \qquad (11)$$

2) Demand balance

The demand that the hydro chain has to satisfy is an input data provided by a hydro-thermal coordination tool. The total production of the hydro chain is computed as the sum of the productions of all its hydro units. In fact, the objective would be to minimize the absolute value of the deviation. However, as the absolute value function is non-linear, we apply the technique to model the absolute value linearly by introducing two additional variables.

$$\sum_{i=1}^{I}\left(p_{ik}\right) - \delta d_k^{+} + \delta d_k^{-} = d_k \qquad \forall k \qquad (12)$$

Note that deviation variables are activated only when power demand is not completely satisfied as they are penalized in the objective function.

3) Initial and final reservoir level

Reservoir level conditions are stated as follows:

$$v_{ik} = v_i^{o} \qquad \forall i, k = 1 \qquad (13)$$

$$v_{ik} = v_i^{f} \qquad \forall i, k = K+1 \qquad (14)$$

4) Water Rights

It is usual in hydro systems the existence of water rights derived from ecological flows, irrigation requirements, etc. They can be modeled as:

$$\underline{\theta}_{ik} \le q_{ik} + s_{ik} \le \overline{\theta}_{ik} \qquad \forall i, \forall k \qquad (15)$$

5) Input-output curve modeling

In order to introduce in the SHSP the input-output curve modeling shown in figure 2, following constraints are included. First of all, (16), (17) and (18) state that when the unit is off, the outflow is zero and when it is on, the outflow must be within the interval [$\underline{q}_{ik}$, $\overline{q}_{ik}$],

$$q_{ik} = u_{ik} \cdot \underline{q}_{ik} + q_{ik}^{a} + q_{ik}^{b} \qquad \forall i, \forall k \qquad (16)$$

$$q_{ik}^{a} \le u_{ik} \cdot \left(q_{ik}^{mxef} - \underline{q}_{ik}\right) \qquad \forall i, \forall k \qquad (17)$$

$$q_{ik}^{b} \le u_{ik} \cdot \left(\overline{q}_{ik} - q_{ik}^{mxef}\right) \qquad \forall i, \forall k \qquad (18)$$

After defining the water discharge physic limits, it is necessary to express the power generation function. In (19) a linear approximation in presented, where the first term deals with the discrete commitment decision and the second and third ones take into account the slopes each linear function, which are multiplied by its corresponding turbine discharge variables.

$$\begin{aligned} p_{ik} = {} & u_{ik} \cdot \underline{p}_{ik} + q_{ik}^{a} \cdot [(p_{ik}^{mxef} - \underline{p}_{ik})/(q_{ik}^{mxef} - \underline{q}_{ik})] \\ & + q_{ik}^{b} \cdot [(\overline{p}_{ik} - p_{ik}^{mxef})/(\overline{q}_{ik} - q_{ik}^{mxef})] \end{aligned} \qquad \forall i, \forall k \qquad (19)$$

6) Logic constraint

This constraint ensures the coherence among the binary variables related to the discrete decisions. For instance, it does not allow to start-up a unit which is already on.

$$y_{ik} + u_{ik-1} - u_{ik} - z_{ik} = 0 \qquad \forall i, \forall k \qquad (20)$$

Note that additional logic constraints could be easily formulated. For instance, sometimes it is necessary to limit the number of start-up and shutdown operations during each day. This could be attained imposing that the sum of binary variables y_{ik} and z_{ik} along the involved periods must be lower than the maximum number of operations allowed.

7) Reservoir capacity limits

The reservoirs management has to take into account their physical capacity limits,

$$\underline{v}_{ik} \le v_{ik} \le \overline{v}_{ik} \qquad \forall i, \forall k \qquad (21)$$

V. STUDY CASE

The hydro scheduling iterative model presented in this paper has been applied to a real size river basin. The model has been implemented in GAMS, using solver CPLEX 7.1 for solving the SHSP corresponding to each iteration.

A. Input Data

The considered hydro chain owns ten cascaded reservoirs and seven hydraulic generating units. Figure 3 shows the complete topology and table 4 summarizes the most relevant characteristic parameters of each unit. With respect to water rights, unit u_2 has to fulfill a maximum outflow of 38 m^3/s during the whole day.

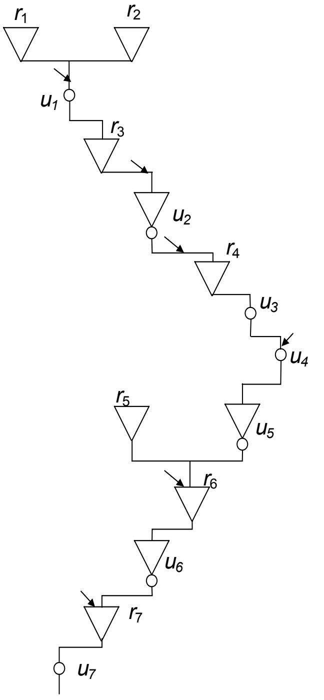


Figure 3: Hydro chain topology

TABLE I
CHARACTERISTICS PARAMETERS

	$\overline{v}$ [Hm^3]	$\underline{v}$ [Hm^3]	$\overline{p}_{c,k}$ [MW]	$\overline{q}_{c,k}$ [m^3/s]	v^0 [Hm^3]	w [m^3/s]	v^f [Hm^3]
u_1	-	-	14.2	11	-	5	
u_2	64.9	5.95	80	62	35.44	1	35.42
u_3	-	-	37	35	-		
u_4	-	-	32	45	-		
u_5	313.1	60.41	72	100	164.5		163.5
u_6	12.7	6	120	124	9.4		8.74
u_7	-	-	30.6	120	-		
r_1	50	7	-	9	20.1		20.1
r_2	20	3	-	11	10.1		10.1
r_3	28.3	.5	-	31	14.4		14.12
r_4	0.49	0.11	-	35	0.3	1	0.3
r_5	1.66	07	-	48	1.18		1.1
r_6	0.83	0.6	-	56.5	0.72	1	0.67
r_7	12.55	7.1	-	120	3.0	12	2.79

The temporal scope is one day divided into 24 hourly periods. Figure 4 shows the hourly demand to be supplied by the whole hydro sub-system, where higher values occur during peak hours in order to minimize the thermal variable cost.

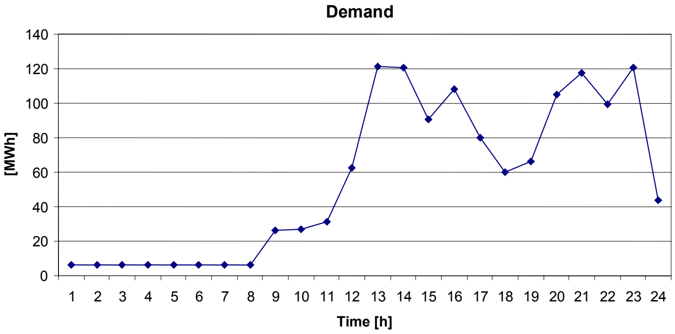


Figure 4: Hourly demand profile

B. Results analysis

The example case was solved for several values of the relaxation parameter α, on a Pentium IV, 1.5 GHz with 256 MB. Figure 5 and Table II present how hydro units fulfill the objective demand without any deviation. The production of u_1 is a good example of a run-of-river unit, which is forced to produce its own natural inflow during the whole day.

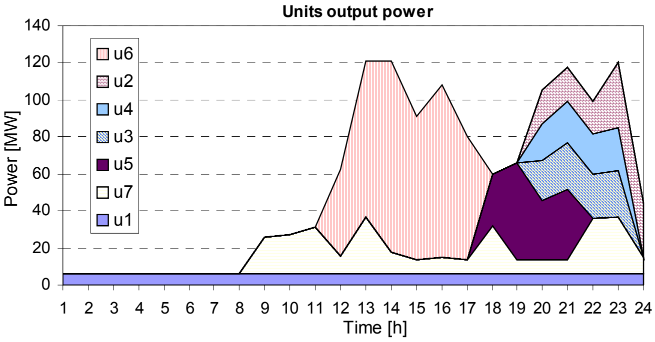


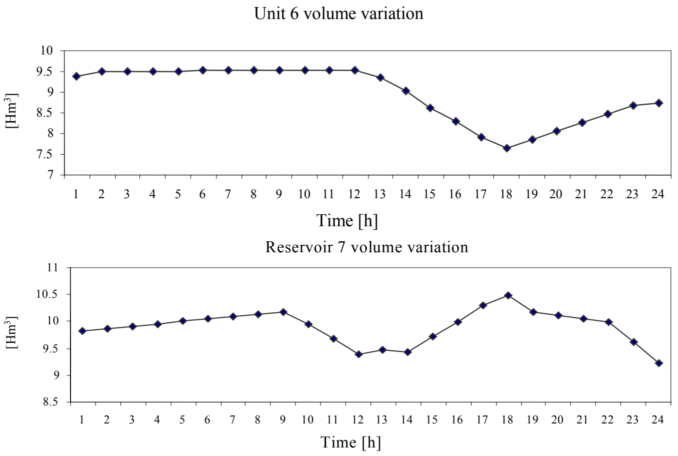
Figure 5: Hydro units productions

TABLE II

HYDRO SCHEDULING OBTAINED

	h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	h11	h12	h13	h14	h15	h16	h17	h18	h19	h20	h21	h22	h23	h24
u1	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4	6.4
u2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	18.1	18.1	18.1	35.9	28.8
u3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.7	25.1	23.7	25.1	0.0
u4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	19.7	22.7	21.5	22.7	0.0
u5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	28.2	52.1	31.7	37.7	0.0	0.0	0.0
u6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	47.2	84.1	103.0	76.9	93.2	66.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
u7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	19.6	20.8	24.7	9.0	30.5	11.5	7.5	8.3	7.5	25.4	7.5	7.5	7.5	29.8	30.5	8.7

Another interesting result is the reservoir management. Figure 6 shows the evolution of the storage water in u_6 and r_7 along the hydro scheduling scope. During the last 4 hours, reservoir u_6 releases the water stored previously in order to satisfy its final volume target. On the other hand, reservoir r_7 behaves in a different way. In the last hours it stores the water previously discharged fulfilling the final level condition.


Unit 6 volume variation

Reservoir 7 volume variation

Figure 6: Reservoir management

Finally, the convergence gap evolution is shown in table II for different α values. For $\alpha = 1$, the execution time was 273 s. Note that the best performance is obtained when updating dynamically the under-relaxation factor.

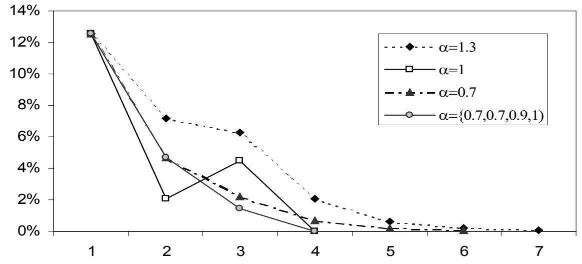


TABLE II

CONVERGENCE ERROR EVOLUTION

	α=1.3	α=1	α=0.7	α={.7,.7,.9,1}
iter1	12.55%	12.55%	12.55%	12.55%
iter2	7.17%	2.07%	4.67%	4.67%
iter3	6.26%	4.48%	2.23%	1.45%
iter4	2.05%	0.00%	0.68%	0.00%
iter5	0.60%	-	0.21%	-
iter6	0.18%	-	0.06%	-
iter7	0.05%	-	-	-

VI. CONCLUSIONS

An under-relaxed iterative procedure has been developed for the short-term hydro-scheduling problem. A simplified problem is solved each iteration, where the power/discharge curves of the head dependent units are determined using previous results. The simplified hydro-scheduling problem is formulated as a mixed-integer linear programming problem where binary variables are introduced to deal with the discrete hydro unit-commitment decisions. The example case results have shown that the model is suitable to schedule real hydro chains. Finally, although the convergence of the procedure cannot be guaranteed from the theoretical point of view, the updating procedure proposed in this paper has lead to satisfactory results in all the cases analyzed.

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