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Stochastic Market Equilibrium Model for Generation Planning

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Abstract—**It is widely accepted that medium-term generation planning can be advantageously modeled through market equilibrium representation. There exist several methods to define and solve this kind of equilibrium in a deterministic way.**

Medium-term planning is strongly affected by uncertainty in market and system conditions, thus extensions of these models are commonly required. The main variables that should be considered as subject to uncertainty include hydro conditions, demand, generating units' failures and fuel prices. This paper presents a model to represent medium-term operation of electrical market that introduces this uncertainty in the formulation in a natural and practical way.

Utilities are explicitly considered to be intending to maximize their expected profits and biddings are represented by means of a conjectural variation. Market equilibrium conditions are introduced by means of the optimality conditions of a problem, which has a structure that strongly resembles classical optimization of hydro-thermal coordination. A tree-based representation to include stochastic variables and a model based on it are introduced.

This approach for market representation provides three main advantages: robust decisions are obtained; technical constraints are included in the problem in a natural way, additionally obtaining dual information; and big size problems representing real systems in detail can be addressed.**Index Terms— Electricity Markets, Game Theory, Stochastic Programming, Hydroelectric-Thermal Power Generation, Conjectural Variation.**

I. Introduction

The new framework of competitive generation markets arisen in a large number of countries, is requiring the development of new operation and planning tools to aid in decision task traditionally made by tools designed for a centralized environment.

One of these strategic tasks is the medium- and long-term hydrothermal coordination that is being commonly addressed in deregulated environments by finding a Nash equilibrium. The equilibrium point is defined as a set of prices, generator outputs, consumption and other relevant numerical quantities, which no market agent could modify unilaterally, by changing its behavior, without a decrease in its profit.

Different methods have been suggested to compute this equilibrium. It is possible to find it by maximizing a profit function for one agent if some restrictive assumptions are accepted [1], [2]. Other alternatives include game theory [3], iterative methods based on a representation of the bidding process [4], or genetic algorithms [5]. As the market equilibrium is mathematically characterized by a set of equations and inequalities that include the so-called complementary conditions, it has been proposed to deal with this issue using techniques as heuristics [6], the linear complementary problem [7], [8], and the inclusion of equilibrium constraints [9]. These approaches are characterized by its complexity.

The focusing used in this paper to solve equilibrium market is different from the previous. The market is represented using a conjectural variation approach. The adjustment of production that generator companies will produce as response to a price variation is assumed as linear and known [10]. It is possible to show that, under some reasonable assumptions, the solution of the market equilibrium conditions is equivalent to the solution of a minimization problem with a structure that strongly resembles classical optimization of hydro-thermal coordination [11].

This paper extends this market equilibrium representation technique to allow the consideration of uncertainty in some of the market conditions. The structure of the optimization problem is suitable to fit in stochastic representations for the operation of hydro-thermal generation systems [12], [13], [14].

Five main sources of uncertainty can be identified when it comes to medium-term generation operation: hydro inflows, fuel prices, system demand, generating units' failures and competence behavior. Any of those factors can be considered as stochastic with the proposed focusing based on a scenario tree representation. The proposed model is specially interesting from a practical point of view, because it is able to compute robust decisions for big size systems including a detailed description of technical characteristic of the system that determine the shape of the agents cost functions.

In section 2, the deterministic case for market equilibrium computation by optimization is summarized and generalized for the stochastic case; section 3 presents a study case that shows the model capabilities and finally the main conclusions are stated in section 4.

II. Stochastic Equilibrium Computation

A. *Deterministic market equilibrium*

In [11] medium-term market equilibrium was defined from a deterministic point of view. It was proved that under some

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reasonable assumptions, it can be formulated as an equivalent minimization problem. This result is summarized herein in a simplified version, for a single unitary period and without contracts signed by generation companies.

Let us assume a generation market in which offers are a linear function of price:

$$P_i = P_i^0 + \alpha_i \cdot \lambda \qquad i = 1,\dots,n \tag{1}$$

P_i is the offered power by the i generation company, λ the demanded price, and P_i^0 and α_i two constants which characterize the linear offer. Let us also assume that demand can be described by the following equation:

$$D = D^0 - \alpha_0 \cdot \lambda \tag{2}$$

D^0 is the intercept of the demand, which is assumed to be known. α_0 represents the demand slope.

Given P_i^0, α_i, D^0 and α_0, the market clearing price λ, generated powers P_i and demand D are computed by solving the system formed by (1), (2) and the generation-demand balance equation:

$$D = \sum_i P_i \tag{3}$$

In order to compute its offer (i.e., the P_i^0 and α_i coefficients) each generation company must make some assumptions on the offers that the other utilities are going to send (i.e., it must have some idea on the coefficients P_j^0 and α_j, $j \neq i$). For each particular assumption on these coefficients, there is a residual demand function (4). Note that the sum of α_j where $i \neq j$ includes α_0 .

$$P_i = D - \sum_i P_i = \left(D^0 - \sum_{j \neq i} P_j^0 \right) - \lambda \cdot \sum_{j \neq i} \alpha_j \tag{4}$$

Given this function, companies' offer must maximize its profit, which is assumed to be:

$$B_i = \lambda(P_i) \cdot P_i - C_i(P_i) \qquad \forall i \tag{5}$$

The first term represents the market income, and the second one the production cost. By differentiating and zeroing:

$$\lambda - \frac{\partial C_i}{\partial P_i} - \frac{1}{\sum_{j \neq i} \alpha_j} \cdot P_i = 0 \qquad \forall i \tag{6}$$

Equations (2), (3) and (6) constitute a Nash market equilibrium that can be obtained from the solution of the minimization problem:

$$\begin{aligned} \min_{P_i, D} \quad & \sum_i \left(\overline{C_i}(P_i) \right) - U(D) \\ s.t. \quad & \sum_i P_i = D \qquad : \lambda \end{aligned} \tag{7}$$

The effective cost functions $\overline{C_i}(P_i)$ are defined as:

$$\overline{C_i}(P_i) = C_i(P_i) + \frac{P_i^2}{2 \cdot \sum_{j \neq i} \alpha_j} \tag{8}$$

The utility function $U(D)$ is defined as:

$$U(D) = \int_0^D \lambda(D) \cdot dD = \frac{1}{\alpha_0} \cdot \left(D \cdot D^0 - \frac{D^2}{2} \right) \tag{9}$$

The clearing price λ is the Lagrange multiplier of the constraint (3).

B. Scenario tree definition

If some of the parameters included in the previous representation of market equilibrium are considered to be subject to uncertainty, the approach has to be extended. A basic alternative is taking it into account by analyzing a set (normally a reduced one, due to the size of the problem) of representative scenarios. Market equilibrium is computed for each of them separately. Results obtained from these scenarios are analyzed together. The main drawback of this method is its lack of robustness for short-term operation: different decisions are obtained for the first time period in each scenario. Besides, the reduced number of scenarios makes the analysis not accurate enough, if treated as a Montecarlo simulation.

A more advanced possibility to include uncertainty, specially when the size of the problem prevents from an extensive Montecarlo analysis is using a scenario tree. It has the main advantage of allowing both to include stochastic variables and to compute a single robust decision for the first period of the study. A sample tree is shown in Fig. 1. Periods are numbered consecutively and branches are also numbered consecutively starting by *b1* within each period.

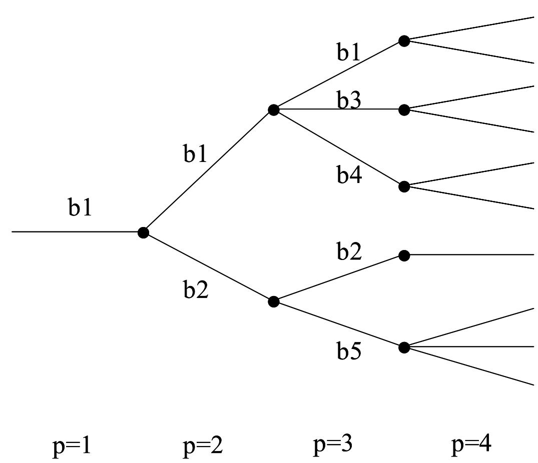

Fig. 1 Sample scenario tree

Tree structure is established using a correspondence and a set. The correspondence *a(p,b)* relations the branch *b* of period *p* with the one that is immediately before it. The set *B(p)* includes all the branches that are defined in period *p*. In the tree of the figure, for example, $a(p_3,b_5)=b_2$ and $B(p_3)=\{b_1, b_2, b_3, b_4, b_5\}$

A probability w_{pb} is defined for each branch. To guarantee tree coherence, the total probability of the branches in a period must add one:

$$\sum_{b \in B(p)} w_{pb} = 1 \quad \forall p \tag{10}$$

Besides, the probability of all the branches following a single one must add the probability of the previous branch:

$$\sum_{b^*/a(p,b^*)=b} w_{pb^*} = w_{p-1,b} \quad \forall p>1 \quad \forall b\in B(p) \tag{11}$$

The use of a scenario tree implies the definition of an objective function to be maximized along the whole tree for every agent in the market. In this paper the mean profit is used defined as:

$$B_i = \sum_p \sum_{b\in R(p)} \left[l_p \cdot w_{pb} \cdot \left[\lambda_{pb} \cdot P_{ipb} - C_{ipb}\left(P_{ipb}\right)\right]\right] \tag{12}$$

This expression includes the duration of each period, l_p. Alternative objective functions could be defined, including those non-neutral to risk modifying the value of probabilities.

This new definition of profit implies that prices, companies' productions, or costs may be different for each branch tree. The use of different cost functions for each branch tree allows a probabilistic representation of any parameter that affects this costs, such as fuel costs, hydro inflows or units' availability.

C. Stochastic market equilibrium definition

The previous market representation requires an extension of market equilibrium concept.

First, offers have different slope in each branch of the tree, representing different behaviors of the companies under different circumstances.

$$P_{ipb} = P^0_{ipb} + \alpha_{ipb} \cdot \lambda_{pb} \qquad i=1,\ldots,n \tag{13}$$

Additionally, demand is also different for each branch that can be used to introduce uncertainty in demand value.

$$D_{pb} = D^0_{pb} - \alpha_{0pb} \cdot \lambda_{pb} \tag{14}$$

$$D_{pb} = \sum_i P_{ipb} \tag{15}$$

Similarly to the deterministic case (6), the maximization of the profit function for each company leads to:

$$\lambda_{pb} - \frac{\partial C_{ipb}}{\partial P_{ipb}} - \frac{1}{\sum_{j\neq i} \alpha_{jpb}} \cdot P_{ipb} = 0 \qquad \forall i \tag{16}$$

Equations (14), (15) and (16) define the stochastic market equilibrium.

D. Stochastic equilibrium computation by optimization

The newly defined equilibrium can be obtained from the solution of the following minimization problem that is a natural extension of the deterministic one.

$$\min_{P_{ipb},D_{pb}} \quad \sum_p \sum_{b\in R(p)} \left[l_p \cdot w_{pb} \cdot \left[\sum_i \left(\overline{C}_{ipb}\left(P_{ipb}\right)\right) - U\left(D_{pb}\right)\right]\right]$$
$$s.t. \quad \sum_i P_{ipb} = D_{pb} \qquad : \eta_{pb} \tag{17}$$

The effective cost functions $\overline{C}_{ipb}\left(P_{pb}\right)$ are defined as:

$$\overline{C}_{ipb}\left(P_{ipb}\right) = C_{ipb}\left(P_{ipb}\right) + \frac{P^2_{ipb}}{2\cdot \sum_{j\neq i} \alpha_{jpb}} \tag{18}$$

The utility function $U(D_{pb})$ is defined as formerly (9):

$$U\left(D_{pb}\right) = \int_0^{D_{pb}} \lambda\left(D\right)\cdot dD = \frac{1}{\alpha_0} \cdot \left(D_{pb} \cdot D^0_{pb} - \frac{{D_{pb}}^2}{2}\right) \tag{19}$$

The clearing price for each period and branch λ_{pb} is now obtained from the Lagrange multiplier of the constraint as follows.

$$\lambda_{pb} = \frac{\eta_{pb}}{l_p \cdot w_{pb}} \tag{20}$$

Optimality conditions of the presented optimisation problem are the same as equilibrium market equations. This can be easily shown for first order conditions. The Lagrange function for the previous optimisation problem is:

$$\begin{aligned} &\mathfrak{L}(P_{ipb}, D_{pb}, \eta_{pb}) = \\ &\sum_p \sum_{b\in R(p)} l_p \cdot w_{pb} \cdot \left[\sum_i \overline{C}_{ipb}\left(P_{ipb}\right) - U\left(D_{pb}\right)\right] + \\ &+ \sum_p \sum_{b\in R(p)} \eta_{psb} \cdot \left(D_{pb} - \sum_i P_{ipb}\right) \end{aligned} \tag{21}$$

The solution to the optimization problem is obtained by differentiating and zeroing this function.

$$\frac{\partial \mathfrak{L}(P_{ipb}, D_{pb}, \eta_{pb})}{\partial \eta_{pb}} = 0 = D_{pb} - \sum_i P_{ipb} \tag{22}$$

$$\begin{aligned} &\frac{\partial \mathfrak{L}(P_{ipb}, D_{pb}, \eta_{pb})}{\partial P_{ipb}} = 0 = \\ &= w_{pb} \cdot l_p \cdot \left(\frac{\partial C_{ipb}(P_{ipb})}{\partial P_{ipb}} - \frac{P_{ipb}}{\sum_{j\neq i} \alpha_{jpb}} \right) - \eta_{pb} \end{aligned} \tag{23}$$

$$\frac{\partial \mathfrak{L}(P_{ipb}, D_{pb}, \eta_{pb})}{\partial D_{pb}} = 0 = \frac{l_p \cdot w_{pb}}{\alpha_0} \left(D^0_{pb} - D_{pb}\right) - \eta_{pb} \tag{24}$$

These three expressions are equivalent to those that define market equilibrium. Equation (22) corresponds to (15), the balance of generation and demand. Equation (23) is equal to (16), the market equilibrium condition, if price is computed using (20). Finally, equation (24), with the same price expression leads to (14): the linear relationship between price and demand. Thus, stochastic market equilibrium can be computed solving an optimisation problem. A further analysis shows that second order conditions are also the same for both formulations.

E. Inelastic demand representation

In some cases, demand can be treated as a known value for each branch and independent of price. If demand is inelastic, it can be easily shown that market equilibrium can be obtained as the solution to a simplified optimisation problem:

$$\min_{P_{ipb}} \quad \sum_p \sum_{b\in R(p)} \left[l_p \cdot w_{pb} \sum_i \left(\overline{C}_{ipb}\left(P_{ipb}\right)\right)\right]$$
$$s.t. \quad \sum_i P_{ipb} = D^0_{pb} \qquad : \eta_{pb} \tag{25}$$

F. Cost functions representation

The structure of the problem allows adding variables and constraints to represent the cost functions and the technical constraints related to it within the same optimization problem.

To illustrate this point, the power system will be considered as a group of thermal and hydro units owned by different companies. The following new variables have to be included:

t_{jpb} Power generation of thermal unit j in branch b of period p.

h_{mpb} Power generation of hydro unit m in branch b of period p.

b_{mpb} Power consumption of pumped-hydro unit m in branch b of period p.

r_{mpb} Energy reservoir level of hydro unit m in branch b at the end of period p. The value for the last period is considered to be known.

s_{mpb} Energy spillage of hydro unit m in branch b of period p.

Generated power, as will be shown later, may be computed as a linear combination of decision variables.

Some additional parameters must also be considered. Hydro inflows have been assumed as stochastic, and so they have a different value in each branch.

$\overline{t_j}$ Maximum power generation of thermal unit j.

δ_j Variable cost of thermal unit j.

o_j Owner utility of thermal unit j.

$\overline{b_m}$ Maximum pumping power consumption of hydro unit m.

$\overline{r_{mp}}$ Maximum energy reservoir storage of hydro unit m in branch at the end of period p.

$\underline{r_{mp}}$ Minimum energy reservoir storage of hydro unit m at the end of period p.

f_{ipb} Run-off-the-river hydro energy for utility i in branch b of period p.

I_{mpb} Hydro inflows (except run-off-the-river) of hydro unit m in branch b of period p.

r_{m0} Initial energy reservoir level of hydro unit m.

ρ_m Performance of pumping for hydro unit m.

o_m Owner utility of hydro unit m.

u_m, v_m Constant and linear term of the relationship between reservoir storage and maximum power for hydro unit m (run-off-the-river hydro energy is not included).

Generated power for utility i in branch b of period p is computed as a linear combination of decision variables.

$$P_{ipb} = \sum_{j/o_j=i} t_{jpb} + \sum_{m/o_m=i} \left(h_{mpb} - b_{mpb}\right) + f_{ipb} \tag{26}$$

The following constraints are added to the problem.

Decision variables bounds:

$$t_{jpb} \le \overline{t_j} \tag{27}$$

$$h_{mpb} \le u_m + r_{m,p-1,b} \cdot v_m \tag{28}$$

$$b_{mpb} \le \overline{b_m} \tag{29}$$

$$\underline{r_{mp}} \le r_{mpb} \le \overline{r_{mp}} \tag{30}$$

Power balance for each branch and period. It is the result of including (26) in (15).

$$\sum_j t_{jpb} + \sum_m h_{mpb} + f_{ipb} = D_{pb} + \sum_m b_{mpb} \tag{31}$$

Energy balance for each period, branch and hydro unit.

$$r_{mpb} - r_{m,p-1,a(p,b)} = -l_b \cdot \left(h_{mpb} - \rho_m \cdot b_{mpb}\right) + I_{mpb} - s_{mpb} \tag{32}$$

And finally, cost function can now included explicitly in the objective function as a linear combination of decision variables, keeping the structure of the optimization problem.

$$C_{ipb} = \sum_j \delta_j \cdot t_{jpb} \tag{33}$$

III. STUDY CASE

A. System description

This study case represents operation in a market divided into nine periods representing January to September. It has been considered that hydro energy storage at the end of September is the lowest in the year and easy to forecast.

Table I shows demand, which has been considered inelastic.

TABLE I DEMAND (GWH)

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep
Dem.	20124	18610	18425	16584	17549	19032	20014	18768	18024

Seven generation companies are considered to be competing in the market. Table II shows its generation structure and Table III the ranges for variable costs. This case is an example of a real size case, and loosely represents Spanish daily market, where a large number of groups is owned by a reduced group of companies. The model could be also used if the same set of groups would be property of a large number of companies.

TABLE II INSTALLED POWER (MW) AND NUMBER (IN PARENTHESES) OF GROUPS OWNED BY GENCOS

Company	Nuclear	Coal	Gas	Hydro	Pumping
1	3459 (3)	5517 (15)	3259 (10)	4043 (7)	1431 (5)
2	3169 (5)	1167 (5)	4879 (11)	5421 (5)	2282 (2)
3	705 (1)	1888 (7)	1130 (3)	1518 (3)	216 (1)
4	155	1488 (5)	381 (1)	314 (1)	115 (1)
5	-	859 (5)	731 (2)	651 (1)	360 (1)
6	-	-	1518 (3)	-	-
7	-	117 (1)	1142 (3)	-	-

TABLE III VARIABLE COST RANGES (/MWH)

Company	Nuclear		Coal		Gas	
	Min	Max	Min	Max	Min	Max
1	0.6	4.8	12.4	24.3	22.0	44.8
2	0.6	4.8	12.4	24.3	22.0	41.8
3	4.0	6.4	15.8	29.6	22.9	44.8
4	4.8	4.8	12.4	26.3	22.9	22.9
5	-	-	12.4	23.1	38.5	41.7
6	-	-	-	-	22.9	23.0
7	-	-	12.4	12.4	22.0	22.9

Conjectural variations have been assigned according to companies' size, ranging from 2.5 to 5.0 for the largest ones (companies 1, 2 and 3), between 1 and 2 for the medium-sized utilities (4, 5 and 6) and with a single value of 0.5 for the smallest one (company 7). Units for this parameter are

((€/MWh)/GW).

Three different scenarios have been considered for hydro inflows: wet, medium and dry with probabilities 0.1, 0.4 and 0.5 respectively. The highest probability for the worst case from the company point of view (dry scenario leads to highest operation cost) represents a non-neutral to risk point of view. Table IV shows total hydro inflows considered for each one.

TABLE IV SCENARIOS TOTAL HYDRO INFLOWS (GWH)

Scenario	Wet	Medium	Dry
Hydro	16138	10345	7329
Run-off-the-river	21443	17211	12446
Total	37581	27556	19775

The structure of the tree is shown in Fig. 2. The value of inflows is the same for the first three months; the rest of the year makes the difference. These three scenarios have been solved separately (considering a deterministic market equilibrium) and also together, solving a stochastic market equilibrium. The objective of this tree structure is to decide operation for the first three months. Differences between deterministic and stochastic approaches will be studied.

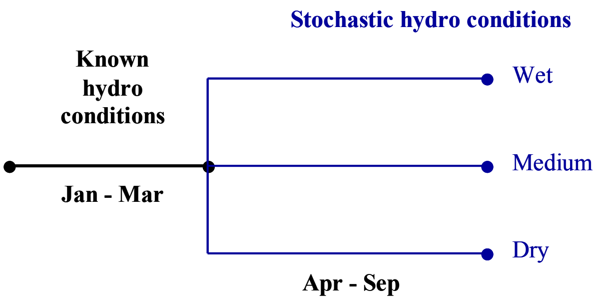

Fig. 2 Scenario tree for study case

The model has been coded in GAMS 21.0 language and solved by using CPLEX 8.1 solver. Execution time for the presented case is about 15 minutes in a 1.7 GHz PC with Pentium IV processor.

B. *Results*

Prices are presented in Fig. 3. Substantial differences appear between stochastic and deterministic analysis. Prices in dry and medium deterministic scenario are similar to those in dry and medium branch of the stochastic case. Shortage of hydro resources prevents companies from different hydro resources management. However, prices in the wet branch of the stochastic case are lower in comparison to deterministic case because more hydro energy is stored in this case. Medium deterministic case and medium stochastic branch are very similar although these similar prices produce different hydro resources operation for the first time periods.

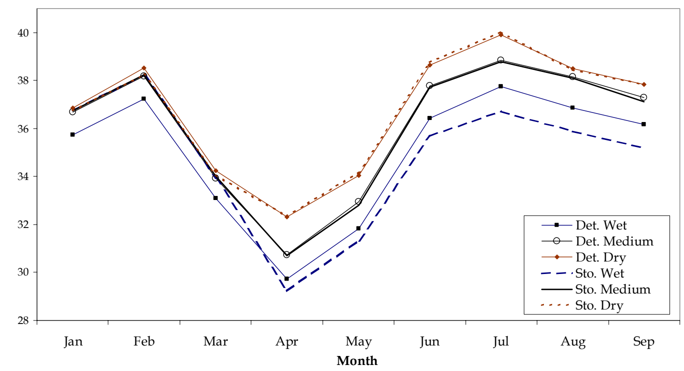

Fig. 3 Prices for deterministic and stochastic cases (€/MWh)

Energy reservoir levels are shown in Fig. 4. For the shake of clarity, deterministic and stochastic reservoir levels have been represented again separately in Fig. 5 and Fig. 6.

The stochastic analysis suggests a more conservative resources management in order to face the dry scenario in case it would happen. In January, February and March a larger amount of hydro energy is reserved in the stochastic case than in the wet and medium deterministic cases.

The analysis of deterministic cases shows that management of dry scenario is unrealistic. High levels of reservoirs levels are kept in order to take advantage of the high prices that will happen from June to September. This would not be a good decision if a wet scenario happened from April (for example) and prices fell down. In a similar way, wet deterministic case is not realistic.

Decisions taken in the stochastic case for wet and dry branches are more robust because in this case, only three months of inflows are supposed to be known in advance.

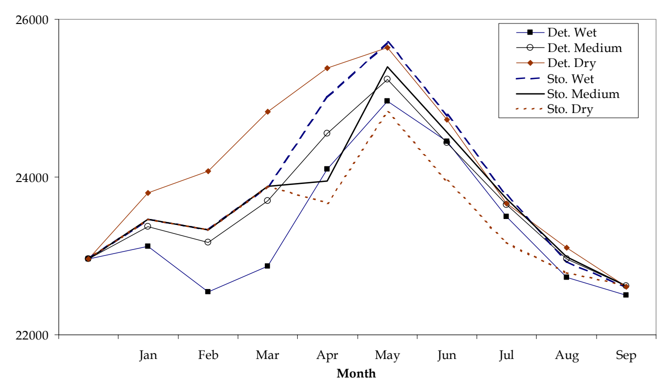

Fig. 4 Reservoir levels for company 2 for deterministic and stochastic cases (GWh)

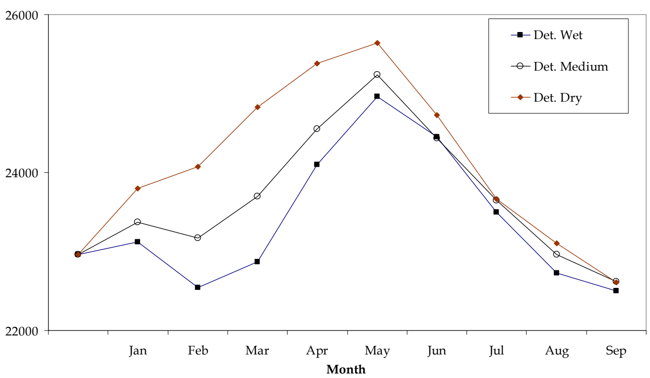

Fig. 5 Reservoir levels for company 2 in deterministic cases (GWh)

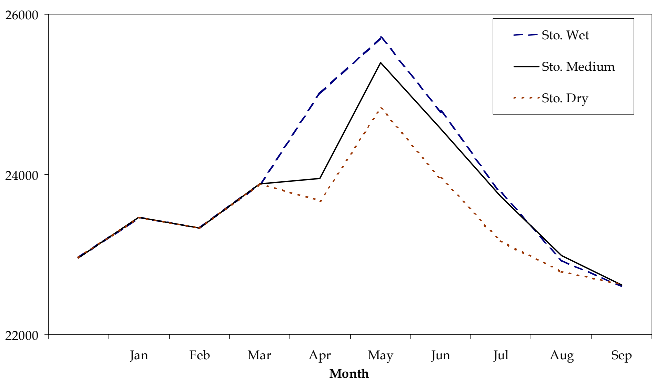

Fig. 6 Reservoir levels for company 2 in stochastic case (GWh)

IV. Conclusions

A stochastic representation of market equilibrium has been introduced, including its equivalence with an optimization problem to solve it efficiently.

Market equilibrium has been represented by means of a conjectural variation, and has been computed along a scenario tree that allows including stochastic variables. A study case with stochastic inflows has been solved and analyzed showing this model advantages. Other variables could have been considered as stochastic using the same method.

The previous features make this approach very interesting from a practical point of view: first, robust decisions are obtained; second, technical constraints are included in the market equilibrium representation in a natural way obtaining dual information; and third, big size problems representing real systems with a large number of companies (or alternatively with a reduced number of companies owning a large number of groups each), can be addressed.

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