

# On Voltage Stability Regions and Voltage Secondary Control

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**Abstract-** This paper presents some new results on the geometry of the reactive load-flow problem. These results have implications in order to assess the system margin to the voltage collapse and to design a secondary voltage control aimed to avoid the voltage collapse. The results are based on certain mathematical and very general properties of the jacobian matrices of the load-flow equations.

**Keywords:** Voltage stability, Voltage collapse, Secondary Voltage Control.

## I. INTRODUCTION

The use of secondary voltage control has been suggested, and even implemented, in order to improve the system security, among other objectives. [1], [2], [3]. However, there is a lack of rigorous theoretical results that establish the limitations of these kind of controls.

On the other hand, one of the most serious stability problems (the voltage collapse) has been related to a static condition: the maximum power transfer point [4]. This condition is, under some assumptions, reached when the load flow jacobian matrix becomes singular.

There have been studies on the effect of secondary voltage control on the voltage collapse problem [5]. The purpose of this paper is to present several theoretical results that establish some characteristics of the maximum power transfer point, and some implications of these results in the design of secondary voltage control.

The secondary voltage control design problem is often addressed by studying a power system model which links the reactive power injections to the voltage magnitudes. The reactive load flows derived from these models have a peculiar structure, allowing application of results from matrix and control theory [6] to the jacobian matrices.

The sequel of this paper is organized as follows: firstly the reactive load-flow model is shown. In the next section, a definition of static stability (linked to the maximum power transfer point and the voltage collapse problem) is introduced, and the jacobian matrices of the previous model analyzed from that point of view. Section 4 deals with the effect of the reactive output limits of the generators. The next section studies the characteristics of the stability region in the space of the control variables (the generators voltages magnitudes). Section 6 shows some implications of

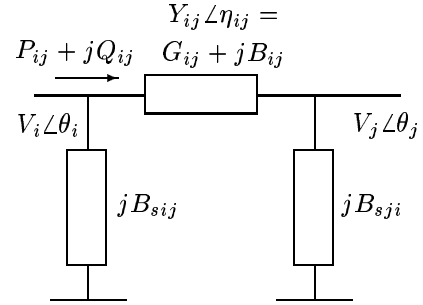


Fig. 1. Line and transformer  $\pi$ -equivalent modeling.

the obtained results in the design of the secondary voltage control. Finally, the paper conclusions are established.

## II. MODELING

This paper addresses the solutions of the reactive load-flow equations. These equations can be obtained as an approximation of the complete set (active and reactive) of load-flow equations. As there are several ways of simplifying these equations, there are also several different sets of reactive load-flow equations. Among them, there can be identified:

### 1. The decoupled load-flow equations:

$$Q_i = \sum_{j \in \mathcal{A}_i} [-B_{sij}V_i^2 - B_{ij}V_i^2 - Y_{ij}V_iV_j \sin(\theta_i - \theta_j - \eta_{ij})] \quad (1)$$

where  $Q_i$  is the injected reactive power at bus  $i$ ,  $\mathcal{A}_i$  is the set of buses connected at bus  $i$ ,  $V_i$  and  $\theta_i$  are the voltage magnitude and phase at bus  $i$ ,  $Y_{ij}$  and  $\eta_{ij}$  are the admittance magnitude and phase of the line or transformer joining buses  $i$  and  $j$ ,  $B_{ij}$  is the imaginary part of that same admittance, and  $B_{si}$  the imaginary part of the shunt admittance attached to bus  $i$  at line or transformer  $ij$  (see figure 1). Note that  $B_{ij} < 0$ .

The decoupled load-flow model treats the voltages phases as given constants. Then, it is obtained:

$$\frac{\partial Q_i}{\partial V_j} = -Y_{ij}V_i \sin(\theta_i - \theta_j - \eta_{ij}) \quad (2)$$

$$\frac{\partial Q_i}{\partial V_i} = \sum_{j \in \mathcal{A}_i} -2(B_{sij} + B_{ij})V_i - Y_{ij}V_j \sin(\theta_i - \theta_j - \eta_{ij}) \quad (3)$$

## 2. The CRIC load-flow equations.

The previous decoupled load-flow model is questionable in view of the following two facts [8]:

(a) close to the boundary of the region of feasible operation (whose characterization is one of the main purposes of this paper) the voltages can be far below 1 p.u.

(b) in some systems, it can be needed to take into account sub-transmission networks whose lines can exhibit low  $X/R$  ratios.

The CRIC model [7] aims to overcome these limitations. In short, from the load-flow equations:

$$P_i = \sum_{j \in \mathcal{A}_i} [G_{ij}V_i^2 - V_iV_jY_{ij} \cos(\theta_i - \theta_j - \eta_{ij})] \quad (4)$$

$$Q_i = \sum_{j \in \mathcal{A}_i} [-B_{si}V_i^2 - B_{ij}V_i^2 - Y_{ij}V_iV_j \sin(\theta_i - \theta_j - \eta_{ij})] \quad (5)$$

it is assumed that the active power flow from bus  $i$  towards bus  $j$

$$P_{ij} = G_{ij}V_i^2 - V_iV_jY_{ij} \cos(\theta_i - \theta_j - \eta_{ij})$$

is a constant. From this equation, it is obtained:

$$\sin(\theta_i - \theta_j - \eta_{ij}) = \pm \frac{\sqrt{Y_{ij}^2V_i^2V_j^2 - (G_{ii}V_i^2 - P_{ij})^2}}{Y_{ij}V_iV_j}$$

Usually  $\eta_{ij} \approx -\frac{\pi}{2}$  y  $\|\theta_i - \theta_j\| \ll \pi/2$ . Therefore, the + sign must be chosen. Then:

$$Q_i = \sum_{j \in \mathcal{A}_i} [-B_{si}V_i^2 - B_{ij}V_i^2 - \sqrt{Y_{ij}^2V_i^2V_j^2 - (G_{ii}V_i^2 - P_{ij})^2} \sin(\theta_i - \theta_j - \eta_{ij})] \quad (6)$$

So

$$\frac{\partial Q_i}{\partial V_i} = -2(B_{si}V_i - \sum_{j \in \mathcal{A}_i} \left[ B_{ij}V_j + \frac{Y_{ij}^2V_j^2V_i - 2G_{ij}V_i^3 + 2G_{ij}P_{ij}V_i}{D_{ij}} \right]) \quad (7)$$

$$\frac{\partial Q_i}{\partial V_j} = -\frac{Y_{ij}^2V_i^2V_j}{D_{ij}} \quad (8)$$

$$D_{ij} = \sqrt{Y_{ij}^2V_i^2V_j^2 - (G_{ij}V_i^2 - P_{ij})^2} \quad (9)$$

In both cases it has been obtained a set of reactive load-flow equations:

$$Q_i = Q_i(V_j) \quad (10)$$

There are, of course, other ways to simplify the load-flow equations in order to isolate the reactive part. It is very

important that in all the cases studied, it is fulfilled that the matrix

$$S = \left[ \frac{\partial Q_i}{\partial V_j} \right] \quad (11)$$

has every non-diagonal element non-positive:

$$S_{ij} \leq 0 \quad i \neq j \quad (12)$$

A matrix with this property is called a Z matrix. Furthermore, it is usual that the diagonal  $S$  elements are positive:

$$S_{ii} > 0 \quad (13)$$

The  $S$  matrix relates the increments in the reactive power injection to the increments in the voltage magnitude. If the PQ buses are denoted by the subscript  $l$  and the PV buses by  $g$ ,  $S$  can be partitioned as:

$$\begin{bmatrix} S_{gg} & S_{gl} \\ S_{lg} & S_{ll} \end{bmatrix} \begin{bmatrix} \Delta V_g \\ \Delta V_l \end{bmatrix} = \begin{bmatrix} \Delta Q_g \\ \Delta Q_l \end{bmatrix} \quad (14)$$

In the sequel, mathematical properties of Z matrices shall be used. There are also very important the mathematical properties of the so-called M matrices. A matrix  $A$  is a M-matrix when its inverse is componentwise positive:

$$A^{-1} \geq 0 \quad (15)$$

In the sequel, the sign  $\geq 0$  ( $\leq 0$ ) applied to a matrix or a vector shall denote that all its elements are non-negative (non-positive).

## III. MAIN RESULTS

When in a power system the demanded reactive power is increased, the bus voltages describe a “nose curve”. The system is considered to be statically stable when the two following conditions are met:

1. An increase in the demanded reactive power in any bus, keeping constant the generators voltages, does not increase the voltage magnitude in any load bus.
2. An increase in the voltage magnitude in any generator, keeping every load constant, does not decrease the voltage magnitude in any load bus.

The first condition states that the system is in the upper side of the “nose curve”. Mathematically, the first condition says that the solution  $\Delta V_l$  from

$$S_{ll}\Delta V_l = \Delta Q_l \quad (16)$$

is non-positive whenever  $\Delta Q_l \leq 0$  (note that they are *injected* reactive power). But this is just a way of stating that  $S_{ll}$  is a M-matrix.

The second condition states that the solution  $\Delta V_l$  of

$$S_{lg}\Delta V_g + S_{ll}\Delta V_l = 0 \quad (17)$$

is non-negative whenever  $\Delta V_g \geq 0$ . As

$$\Delta V_l = -S_{ll}^{-1}S_{lg}\Delta V_g \quad (18)$$

As  $S_{lg} \leq 0$ ,  $S_{ll}$  should be a M-matrix. So, if  $S$  is a Z-matrix, the power system is statically stable if and only if  $S_{ll}$  is a M-matrix.

On the other hand,  $S_{ll}$  is a Z-matrix. Then, it is possible to write:

$$S_{ll} = \alpha I - B \quad (19)$$

where the real number  $\alpha > 0$ ,  $I$  is the identity matrix and  $B \geq 0$ . Then, the following facts can be proved:

1.  $S_{ll}$  is a M-matrix if and only if

$$\alpha > \rho(B) \quad (20)$$

where  $\rho(B)$  is the  $B$  spectral radius [6, p. 21]. This fact allows to easily check if  $S_{ll}$  is a M-matrix, as there are very efficient ways of computing the spectral radius.

2.  $S_{ll}$  is a M-matrix if and only if the real part of every eigenvalue is positive [6, p. 21]:

$$\Re(\lambda) > 0 \quad (21)$$

3. As  $B \geq 0$ ,  $\rho(B)$  is a  $B$  eigenvalue. Therefore,  $\alpha - \rho(B)$  is a  $S_{ll}$  eigenvalue [6, p. 16].

4. As  $B \geq 0$ , there are a right eigenvector  $\mathbf{v}^* \geq \mathbf{0}$  and a left one  $\mathbf{w}^* \geq \mathbf{0}$  corresponding to the eigenvalue  $\rho(B)$  [6, p. 16].

To further proceed, the following complex-plane regions are considered:

- The circle  $\mathcal{C}_B$  is defined as:

$$\mathcal{C}_B = \{z, \|z\| \leq \rho(B)\} \quad (22)$$

Note that, by the spectral radius definition, every  $B$  matrix eigenvalue is in  $\mathcal{C}_B$ .

- The circle  $\mathcal{C}_S$  is defined as the image of  $\mathcal{C}_B$  under the map  $z \rightarrow \alpha - z$ . Note that because of fact 3, every  $S_{ll}$  eigenvalue is in  $\mathcal{C}_S$ .

Now, the main results can be stated:

*Theorem 1:*  $S_{ll}$  is an M-matrix if and only if  $\mathcal{C}_S$  is included in the right complex half-plane.

*Proof:* If  $\mathcal{C}_S$  has non-empty intersection with the left complex half-plane or the imaginary axis, its leftmost point  $\alpha - \rho(B)$  must be non-positive. But as  $\alpha - \rho(B)$  is a  $S_{ll}$  eigenvalue, there is a non-positive  $S_{ll}$  eigenvalue. Because of fact 2,  $S_{ll}$  is not a M-matrix. Therefore, if  $S_{ll}$  is a M-matrix,  $\mathcal{C}_S$  yields in the right half-plane.

To prove the converse statement, assume that  $\mathcal{C}_S$  is included in the right half-plane. Then, every  $S_{ll}$  eigenvalue is in the right half-plane. So, the real part of every  $S_{ll}$  is positive and therefore, because of fact 2,  $S_{ll}$  is a M-matrix. ■

*Theorem 2:* If the matrix  $S_{ll}$  is a continuous function of a parameter  $t > 0$ ,  $S_{ll}(t)$  is a M-matrix in the interval  $t \in [0, t_c)$ , and  $S_{ll}(t)$  is not a M-matrix for  $t = t_c$ , then  $S_{ll}(t_c)$  is a singular matrix.

*Proof:* As  $S_{ll}(t)$  are Z-matrices, it is possible to consider continuous quantities  $\alpha(t)$  and  $B(t)$  such that  $S_{ll}(t) = \alpha(t)I - B(t)$ . As the spectral radius is a continuous function,

$\rho(B(t))$  is a continuous function of  $t$ . Therefore, the circle  $\mathcal{C}_S(t)$  depends continuously on  $t$ .

$S_{ll}(t)$  is a M-matrix if and only if  $\mathcal{C}_S(t)$  is in the right half-plane. As  $\mathcal{C}_S(t)$  is centered on the real axis, it can only get out of the right half-plane by touching the origin. That means that its leftmost point  $\alpha(t) - \rho(B(t))$  is 0 when  $t = t_c$ . But as this point is a  $S_{ll}$  eigenvalue,  $S_{ll}$  is singular when  $t = t_c$ . ■

From a more physical point of view, the last statement means that a power system becomes statically unstable by a singularity in the power-flow jacobian, i.e., by a voltage collapse. The parameter  $t$  can be considered as the time, or any other parameter of interest, according which the operating point and the jacobian is changing. So long as no generator hits limits, the jacobian can be considered to change in a continuous way. The effect of generator limits is considered in the following section.

Additionally, it has been proven that the eigenvector associated to the voltage collapse (the eigenvector associated to the eigenvalue  $\alpha - \rho(B)$ ) is componentwise non-negative. This statement follows immediately from fact 4.

#### IV. GENERATORS LIMITS

The purpose of this section is to study the effect that the generators limits have on the power system static stability. The usual way of modeling these limits is to assume that the generators are no longer PV buses when its reactive output reaches a maximum value, but they become PQ buses injecting that maximum reactive power.

Therefore, when a generator hits limits, the  $S_{ll}$  matrix transforms in a new matrix  $S'_{ll}$  by adding a new row and a new column for the new PQ bus. So, it can be written

$$S'_{ll} = \begin{bmatrix} S_{ll} & S_{lg} \\ S_{gl} & S_{gg} \end{bmatrix} \quad (23)$$

Of course  $S_{lg} \leq \mathbf{0}$  and  $S_{gl} \leq \mathbf{0}$ . Then, it can be proven that the minimum  $S'_{ll}$  eigenvalue is lesser than the minimum  $S_{ll}$  eigenvalue. In order to prove this result, let us firstly to prove the following

*Claim 1:* Let us assume that  $B$  is a  $n \times n$  componentwise non-negative matrix, and  $B'$  a  $(n+1) \times (n+1)$  componentwise non-negative matrix of the form

$$B' = \begin{bmatrix} B & \mathbf{a} \\ \mathbf{b}^T & c \end{bmatrix} \quad (24)$$

Then,  $\rho(B') \geq \rho(B)$ .

*Proof:* As  $B' \geq 0$ ,  $\mathbf{a} \geq \mathbf{0}$ ,  $\mathbf{b} \geq \mathbf{0}$  and  $c \geq 0$ . Let us consider the parameter-dependent family  $B(\epsilon)$  defined as:

$$B(\epsilon) = \begin{bmatrix} B & \epsilon \mathbf{a} \\ \epsilon \mathbf{b}^T & \epsilon c \end{bmatrix} \quad (25)$$

$\epsilon$  is real number in the interval  $[0, 1]$ . Therefore  $B(\epsilon) \geq 0$ . Because of fact 3,  $\rho(B(\epsilon))$  is a  $B(\epsilon)$  eigenvalue with left and right eigenvectors  $\mathbf{v}_\epsilon^*$  and  $\mathbf{w}_\epsilon^*$ . Because of the well-known sensitivity eigenvalue formula

$$\frac{d\rho(B(\epsilon))}{d\epsilon} = \mathbf{w}_\epsilon^{*T} \frac{dB(\epsilon)}{d\epsilon} \mathbf{v}_\epsilon^* = \mathbf{w}_\epsilon^{*T} \begin{bmatrix} 0 & \mathbf{a} \\ \mathbf{b}^T & c \end{bmatrix} \mathbf{v}_\epsilon^* \geq 0 \quad (26)$$

The last inequality follows from fact 4 and from that  $\mathbf{a} \geq \mathbf{0}$ ,  $\mathbf{b} \geq \mathbf{0}$  and  $c \geq 0$ . Therefore,  $\rho(B(1)) \geq \rho(B(0))$ . But as  $\rho(B(1)) = \rho(B')$  and  $\rho(B(0)) = \rho(B)$ , the claim is proven. ■

Now, the main result of this section can be proven:

*Theorem 3:* Given the above defined matrices  $S_{ll}$  and  $S'_{ll}$ , the minimum  $S'_{ll}$  eigenvalue is smaller than the minimum  $S_{ll}$  eigenvalue.

*Proof:* As both  $S_{ll}$  and  $S'_{ll}$  are Z-matrices, it can be written:

$$S_{ll} = \alpha I - B \quad (27)$$

$$S'_{ll} = \alpha I - B' \quad (28)$$

$\alpha > 0$  is the same number in both equations,  $B \geq 0$  and  $B' \geq 0$ . Note that

$$\begin{aligned} B' &= \alpha I - S'_{ll} = \begin{bmatrix} \alpha I - S_{ll} & -S_{lg} \\ -S_{gl} & \alpha - S_{gg} \end{bmatrix} \\ &= \begin{bmatrix} B & -S_{lg} \\ -S_{gl} & \alpha - S_{gg} \end{bmatrix} \end{aligned} \quad (29)$$

So  $B$  and  $B'$  fulfill the claim's conditions. Therefore  $\rho(B') \geq \rho(B)$ , and

$$\lambda_{\min}(S'_{ll}) = \alpha - \rho(B') \leq \alpha - \rho(B) = \lambda_{\min}(S_{ll}) \quad (30)$$

which proves the theorem. ■

The physical consequence of this theorem is that the static stability of the power system can not be improved by a generator hitting its reactive output limits. This is because the minimum and critical  $S_{ll}$  eigenvalue can only decrease if a PV bus is transformed in a PQ bus<sup>1</sup>.

## V. GEOMETRY OF THE STABILITY REGIONS

As the power system load changes, the power-flow jacobian also changes. It is therefore possible that the power system becomes statically unstable. Then, the generators voltages setpoints can be changed in order to keep the system stable, either manually or by some automatic device such as the voltage secondary control. In control theory terminology, the loads  $\mathbf{Q}_l$  are perturbations and the generators voltages  $\mathbf{V}_g$  are the control variables.

The aim of this section is to describe the geometry of the static stable region when the load is constant, in the  $\mathbf{V}_g$  space. The knowledge of the qualitative features of this region can provide valuable insight in order to design a voltage control or to choose a strategy.

As getting out generators is always detrimental to the system security, it is obvious that the best strategy is to keep every generator below its reactive power limit, even in a marginal amount.

Therefore, the stability region is considered to be bounded by three kind of surfaces:

<sup>1</sup>This fact has been stated in a more general context in [10], although the proof therein provided is incorrect.

1. The surface beyond which  $S_{ll}$  is no longer a M-matrix, i.e., the singular surface where  $\alpha - \rho(B) = 0$ .
2. The surface where the generators hit their reactive limit.
3. The surface where the generators hit their voltage setpoints limits.

The geometry of the third surface is trivial: they are just hyperplanes in the  $\mathbf{V}_g$  space. However, the geometry of the two first surfaces is quite more interesting.

Let us consider the load flow equations

$$\mathbf{Q}_l(\mathbf{V}_l, \mathbf{V}_g) - \mathbf{Q}_l = \mathbf{0} \quad (31)$$

$\mathbf{V}_l$  are the variables, the vector function  $\mathbf{Q}_l(\cdot, \cdot)$  is the reactive power going out from bus  $l$ ,  $\mathbf{Q}_l$  is the reactive power injected in bus  $l$  (a vector of constants), and  $\mathbf{V}_g$  the generators voltages setpoints (a vector of parameters). It is a well-know fact that generically (i.e., almost ever) the so called saddle-node bifurcation (the merging and disappearance of two solutions of the load-flow equations) is met when the load-flow jacobian becomes singular. But

$$\frac{\partial \mathbf{Q}_l}{\partial \mathbf{V}_l} = S_{ll} \quad (32)$$

The saddle-node bifurcation is usually considered to be the mathematical counterpart of the voltage collapse. Therefore, the first surface is nothing else that the voltage collapse surface, in the considered approximation.

Near the singularity,  $S_{ll}^{-1}$  is dominated by the smallest  $S_{ll}$  eigenvalue. That is

$$S_{ll}^{-1} \approx \frac{1}{\alpha - \rho(B)} \mathbf{v}^* \mathbf{w}^{*T} \quad (33)$$

where  $\mathbf{v}^*$  and  $\mathbf{w}^*$  are the eigenvectors introduced in fact 4.

It can also be proven [9], that in parameter ( $\mathbf{V}_g$ ) space the normal vector to the singular surface is

$$\mathbf{v}_G = \mathbf{w}^{*T} \frac{\partial \mathbf{Q}_l}{\partial \mathbf{V}_g} = \mathbf{w}^{*T} S_{lg} \quad (34)$$

As  $\mathbf{w}^* \geq \mathbf{0}$  and  $S_{lg} \leq 0$ , then  $\mathbf{v}_G \leq \mathbf{0}$ . As this happens in every point of the singular surface, the singular surface cannot be very convoluted, but a relatively "smooth" one.

Regarding the second surface (the generators reactive limits), let us note that, from the load flow equations:

$$\begin{bmatrix} S_{gg} & S_{gl} \\ S_{lg} & S_{ll} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{V}_g \\ \Delta \mathbf{V}_l \end{bmatrix} = \begin{bmatrix} \Delta \mathbf{Q}_g \\ \Delta \mathbf{Q}_l \end{bmatrix} \quad (35)$$

In the case of constant load,  $\Delta \mathbf{Q}_l = \mathbf{0}$ . Therefore,

$$(S_{gg} - S_{gl} S_{ll}^{-1} S_{lg}) \Delta \mathbf{V}_g = \tilde{S}_{gg} \Delta \mathbf{V}_g = \Delta \mathbf{Q}_g \quad (36)$$

If the system is statically stable, the non-diagonal elements of  $\tilde{S}_{gg}$  are non-positives. In fact, both non-diagonal elements of  $S_{gg}$  and  $-S_{gl} S_{ll}^{-1} S_{lg}$  are nonpositives ( $S_{ll} \geq 0$  because is a M-matrix, and as  $S_{lg} \leq 0$  and  $S_{gl} \leq 0$ , therefore  $-S_{gl} S_{ll}^{-1} S_{lg} \leq 0$ ).

On the other hand, the  $\tilde{S}_{gg}$  diagonal elements can be positive or negative. It is expected that, far away from

the singularity, they are positives. However, when close to the singular surface, the matrix  $S_{ll}^{-1}$  becomes very big and dominated by the smallest  $S_{ll}$  eigenvalue, and it can be written:

$$\tilde{S}_{gg} \approx -S_{gt}S_{ll}^{-1}S_{lg} \approx \frac{-1}{\alpha - \rho(B)}S_{gt}\mathbf{v}^*\mathbf{w}^{*T}S_{lg} \quad (37)$$

As  $\mathbf{v}^* \geq \mathbf{0}$ ,  $\mathbf{w}^* \geq \mathbf{0}$ ,  $S_{lg} \leq 0$ ,  $S_{gt} \leq 0$  and  $\alpha - \rho(B) > 0$ , it is fulfilled that  $\tilde{S}_{gg} \leq 0$ . In particular, the diagonal elements are non-positives, and possibly negatives.

Note that the  $i$ -th  $\tilde{S}_{gg}$  row is the orthogonal vector to the surface  $Q_i = \text{constant}$ , in the  $\mathbf{V}_g$  space. This is because, when moving on that surface, it must be locally fulfilled that  $\Delta Q_i = 0$ . Then, the previous statement directly follows from equation (36).

Near the singularity, the approximation (37) holds. But that means that each  $\tilde{S}_{gg}$  row is proportional to  $\mathbf{w}^{*T}S_{lg}$ . On the other hand, this is the vector normal to the singular surface  $\mathbf{v}_G$ . In other words: the constant reactive output surfaces are tangents to the singular surfaces.

## VI. IMPLICATIONS FOR THE VOLTAGE CONTROL

From the previous results, it is possible to establish some results that can guide the design of a secondary voltage controller. In particular:

1. It is important to keep the generators from reaching their maximum reactive output levels. In fact, it has been proven that when a generator reaches its limits, the static stability cannot improve (equation (30)).
2. It is always worthwhile to increase the generators voltages in order to avoid the singular surface. This is because the normal vector to that surface  $\mathbf{v}_G$  (defined in equation (34)) is non-positive.
3. It is particularly easy to evaluate the margin to the collapse for the reactive power load-flow. This is because the  $S_{ll}$  minimum eigenvalue, and its eigenvectors, can be computed from the maximum eigenvalue of matrix  $B$ . This can be efficiently done by the power algorithm. This result could be important in order to implement a real-time secondary voltage control.

There are some other stability issues which have not addressed in these paper. In particular, the eigenstructure of the matrix  $\tilde{S}_{gg}$  could be important. This matrix is useful in the stability analysis of the secondary voltage control. As proven in the previous section,  $\tilde{S}_{gg} \leq 0$  near the singularity. So, it has a real negative eigenvalue. On the other hand, it can be a M-matrix under low load conditions. That implies that all its eigenvalues yield in the right half-plane. Therefore, there is the possibility of a zero eigenvalue when getting close to the singular surface. That could cause stability problems.

## VII. CONCLUSIONS

In this paper some characteristics of the voltage collapse point have been derived by using some general properties of the reactive load-flow equations. Some of these properties have been previously detected in simulations [10],

although the theoretical reason was lacking. These properties justifies some current practices in design secondary voltage controllers.

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## IX. BIOGRAPHY

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