On oligopolistic electricity market equilibria and contingent claims

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Abstract

This paper deals with oligopolistic electricity market equilibria when strategic agents can trade in contingent claims, and more specifically options, markets. From the theoretical point of view, new results regarding existence and computation of these equilibria are presented. From the policy point of view, relevant questions regarding regulatory issues are arisen. In the particular case of the Spanish electricity market, the relevant ”options” can include fuel subsidies and certain aspects regarding Costs To Transition payments.

Keywords: Electricity markets, oligopolies, equilibrium computations

1 Introduction

Deregulation processes have created a number of electricity markets around the world. Most of these markets, at present rather immature, are oligopolistic ones, due to the small number of utilities compounded with the un-elastic nature of the electricity demand [1] and, in some cases, constraints in the transmission network [2]. The modelling of these kind of markets in oligopolistic settings have been object of certain attention [3, 4, 5, 6, 7, 8].

The existence of electricity markets have also induced the creation of derivatives markets, mostly futures markets. The existence of these kind of markets is considered to be of outstanding importance in order to have efficient electricity markets. Furthermore, in addition to futures markets, there are also some markets dealing with options and other contingent claims. The behaviour of these markets have been usually studied in the perfect competition case, when standard results of economic theory can be applied [9, 10].

In contrast, this paper is focused in the oligopolistic case. The main result is that the existence of (pure strategies) market equilibria can be assessed when the oligopolistic agents are net writers of call options (or net holders of put options). Otherwise, there are cases when no equilibria exists.

In order to prove this statement, some results pertaining the equivalence between oligopolistic market equilibria and a class of optimization problems have been used. These results have been extended in the paper, in a way resembling (but not identical to) Lagrangian relaxation theorems to include a quite general class of contingent claims. The relevant mathematical theory includes certain results regarding variational problems.

The importance of these results is not limited to the options markets case used to motivate the study. For instance, in the Spanish power system, coal subsidies and regulations regarding the sharing of CTCs (Cost of Transition to Competence) between utilities in function of the yearly system behaviour can be modelled by using ”options” as the ones studied in the paper. Therefore, the presented results can shed light of the actual utilities' behaviour and provide policy guides to the Regulator.

2 Variational inequalities and oligopolistic equilibria

Let us assume that a one-to-many map $\mathbf{F}(\mathbf{x})$ is defined from a subset $K \in \Re^n$ to $\Re^n$. The variational inequality problem $\mathrm{VI}(\mathbf{F}, K)$ is to determine a vector $\mathbf{x}^* \in K$ such that

$$\langle \mathbf{F}^*, \mathbf{x} - \mathbf{x}^* \rangle \geq 0, \quad \mathbf{F}^* \in \mathbf{F}(\mathbf{x}^*), \forall \mathbf{x} \in K \tag{1}$$

where $\langle \mathbf{a}, \mathbf{b} \rangle$ denotes the $\mathbf{a}$ and $\mathbf{b}$ scalar product [11].

To model the oligopolistic equilibrium conditions, the following problem is proposed. It is assumed that there is just one period. Anyway, generalization to multiperiod problems is immediate, although it will not be addressed in this note.

In equilibrium, each utility u maximizes the net income I_u, that is, the difference between income and cost.

$$I_u = R_u - C_u \tag{2}$$

The revenue is computed as the price times the sold power, i.e., the generated power P_u minus the power sold either by contracts for differences D_u or other derivatives ϕ_u:

$$R_u = \pi \left(P_u - D_u - \phi_u \right) \tag{3}$$

The power sold under derivatives contracts is a function of the clearing price π

$$\phi_u = \phi_u(\pi) \tag{4}$$

Finally, it is needed to assume some relationship between the market price π and the generated power P_u (the residual demand curve)

$$\pi = \pi_u(P_u) \tag{5}$$

Therefore, the utility u tries to solve the problem

$$\max_{P_u, R_{Ou}} \pi_u(P_u) \left(P_u - D_u - \phi_u \right) - C_u(P_u) \tag{6}$$

In order to complete the model, the conjectural price equation must be stated. In the sequel, a linear relationship will be assumed, so that

$$\frac{\partial \pi_u}{\partial P_u} = -a_u \tag{7}$$

being $a_u > 0$ a known constant. Besides, the system demand D must be met:

$$\sum_u P_u = D \tag{8}$$

Very often, derivative contracts are call or put options. Formally, these derivatives are characterized by

$$\text{(call) } \phi_u = \begin{cases} 0 & \text{if } \pi < \pi_{cu} \\ \overline{\phi_u} & \text{if } \pi > \pi_{cu} \end{cases} ; \text{(put) } \phi_u = \begin{cases} 0 & \text{if } \pi > \pi_{cu} \\ \overline{\phi_u} & \text{if } \pi < \pi_{cu} \end{cases} \tag{9}$$

being $\overline{\phi_u}$ the contracted power and π_{cu} the strike price: the two parameters that define the contract.

It can be shown that the above model can be stated as a variational inequality problem VI$(\mathbf{F}(\mathbf{x}), K)$, where

$$\mathbf{x} = \begin{bmatrix} P_1 \\ \vdots \\ P_U \\ \phi_1 \\ \vdots \\ \phi_U \\ \pi \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} \partial_{P_1} C_1 + a_1 (P_1 - D_1 - \phi_1) - \pi \\ \vdots \\ \partial_{P_U} C_U + a_U (P_U - D_U - \phi_U) - \pi \\ \nu_1(\pi, \pi_{c1}) \\ \vdots \\ \nu_U(\pi, \pi_{cU}) \\ D - \sum_u P_u \end{bmatrix}$$

$$K = \left\{ \phi_u \setminus 0 \le \phi_u \le \overline{\phi_u} \right\} \tag{10}$$

ν_u is an indicatrix function of the corresponding option. That is, it is a continuous function such that $\nu_u(\pi) > 0$ if $\phi_u = 0$, $\nu_u(\pi) < 0$ if $\phi_u = \overline{\Phi_u}$ and $\nu_u(\pi) = 0$ if the value of ϕ_u is undetermined. In the case of call options, $\nu_u(\pi, \pi_{cu}) = \pi_{cu} - \pi$. For a put option, $\nu_u(\pi, \pi_{cu}) = \pi - \pi_{cu}$.

3 Gap theorem and the gap function

The following theorem is fundamental for further development:

Theorem: Let be $\mathbf{F}$ a continuous mapping, and K a non empty, closed and convex set. Let us define $\tilde{\mathcal{L}} : K \times K \longrightarrow \Re$ as:

$$\tilde{\mathcal{L}}(\mathbf{x}, \mathbf{y}) = f(\mathbf{x}) - f(\mathbf{y}) + [\mathbf{F}(\mathbf{x}) - \nabla f(\mathbf{x})]^T (\mathbf{x} - \mathbf{y})$$

being $f : \Re^n \longrightarrow \Re \cup \{+\infty\}$ a convex, lower semicontinouos and differentiable function on K. Let be

$$\tilde{G}(\mathbf{x}) = \sup_{\mathbf{y} \in K} \tilde{\mathcal{L}}(\mathbf{x}, \mathbf{y}), \quad \mathbf{x} \in K$$

Then:

1. $\tilde{G}(\mathbf{x})$ is lower semicontinuous $\forall \mathbf{x} \in K$.
2. $\tilde{G}(\mathbf{x}) \ge 0, \quad \forall \mathbf{x} \in K$.

3. $\tilde{G}(\mathbf{x}^*) = 0 \Longleftrightarrow \mathbf{x}^*$ solves $\mathrm{VI}(\mathbf{F}(\mathbf{x}), K)$.

Proof: See [12].

Although in power system applications function **F** is piecewise continuous (because of discontinuities in the marginal cost), the theorem holds as well. So, let us select as function f:

$$f(\mathbf{x}) = \sum_u C_u(P_u) + \frac{1}{2} a_u \left(P_u - D_u\right)^2 \tag{11}$$

Then, it can be proved, after some manipulation, that

$$\min_{\mathbf{x}\in K} \tilde{G}(\mathbf{x}) = \min_{\pi, 0 \le \phi_u \le \overline{\phi_u}} G(\phi_u, \pi) \tag{12}$$

where $G(\phi_u, \pi) = Z(\phi_u, \pi) + W(\phi_u, \pi)$, and

$$\begin{aligned} Z(\phi_u, \pi) &= \left(\begin{array}{ll} \min_{P_u} & \sum_u C_u(P_u) + \frac{1}{2} a_u \left(P_u - D_u - \phi_u\right)^2 \\ \text{s.t.} & \sum_u P_u = D \end{array} \right) - && (13) \\ & \min_{Q_u} \left\{ \sum_u \left[C_u(Q_u) + \frac{1}{2} a_u \left(Q_u - D_u - \phi_u\right)^2 \right] + \pi \left[D - \sum_u Q_u \right] \right\} \\ W(\phi_u, \pi) &= \sum_u \left[\nu_u(\pi)\phi_u - \nu_u^-(\pi)\overline{\phi_u} \right] && (14) \end{aligned}$$

where

$$\nu_u^-(\pi) = \min\left(\nu_u(\pi), 0\right) \tag{15}$$

The function $Z(\phi_u, \pi)$ is nothing else that the duality gap between the primal (P_u) and dual (Q_u) programmes. Therefore, the following facts can be stated, stemming directly from duality theory:

1. $Z(\phi_u, \pi) \ge 0$.
2. Let be $\lambda(\phi_u)$ the primal problem demand equilibrium multiplier for given ϕ_u. Then $Z(\phi_u, \lambda(\phi_u)) = 0$.
3. $Z(\phi, \pi)$ is convex in π.
4. If $P_u^*(\phi_u)$ and $Q_u^*(\phi_u, \pi)$ are the solutions of the primal and dual programmes, then

$$\partial_\pi Z \ni \sum_u \left(Q_u^* - P_u^*\right)$$

The gradient with respect to ϕ_u can be also easily computed. It happens to be

$$\partial_{\phi_u} Z \ni a_u \left(Q_u^* - P_u^*\right) \tag{16}$$

Regarding function W, it is easy to prove that $W(\phi_u, \pi) \ge 0$ in the optimization dominion $0 \le \phi_u \le \overline{\phi\_u}$. Its gradients are very easy to compute, given its simple analytical form. It can be also proved that $0 \in \partial_u W_u$ if and only if $W_u = 0$.

However, in the general case, function W_u lacks any further “attractive” (convexity, monotonicity ...) properties.

4 Unique equilibrium and “call-like” options

In any case, $G = W + Z \geq 0$, and $G = 0$ means that the equilibrium conditions are fulfilled. So, minimization of G is equivalent to equilibrium computation. An interesting question, both from the theoretical and practical point of view, is to ask if any local minimum is a goal one as well.

In the general case, the answer is a negative one. However, more definite statements can be done if the following “call-like” condition is fulfilled:

Definition: W_u is “call-like” if and only if, in the feasible dominion,

$$(\partial_\pi W_u)\,(\partial_{\phi_u} W_u) \leq 0 \tag{17}$$

Then, the following theorem can be proved

Theorem: If every W_u enjoys the “call-like” property, the G minima fulfils $G = 0$. Furthermore, the minima set is a connected one, possibly comprising a single point.

5 Policy implications

It should be expected that the Regulator aims to get a predictable behaviour of the markets’ agents, which can be formalized as stating that the market equilibrium should be unique and well-defined. Prices caps can be modelled as “call-like” options, and therefore do not compromise this objective. On the other hand, “price floors” or, as in the Spanish regulations, linked “prices caps”, are not “call-like”, and it should be expected multiple equilibria and strategic stochastic behaviour to appear.

In the Spanish regulation, CTCs payments can be considered as a price cap device. However, the sharing of the CTCs payments among the utilities (mainly Endesa and Iberdrola) is contingent on the power actually produced by them, taking two distinct possible values. These provisions are extremely “un-call-like”.

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