8th International Conference on Probabilistic Methods Applied to Power Systems, Iowa State University, Ames, Iowa, September 12-16, 2004

Application of Possibility Theory to Robust Cournot Equilibrium in Electricity Market

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***Abstract*—It is known that Cournot game theory has been one of the theoretical approaches more used to model electricity market behavior. Nevertheless, this approach is highly influenced by the residual demand curves of the market agents, which are usually not precisely known. This imperfect information has normally been studied with Probability Theory, but Possibility Theory might sometimes be more helpful in modeling, not only uncertainty, but also imprecision and vagueness. In this paper, two dual approaches are proposed to compute a robust Cournot equilibrium, when the residual demand uncertainty is modeled with possibility distributions. Additionally, it is shown that these two approaches can be combined into a bi-criteria programming model, which can be solved with an iterative algorithm. Some interesting results for a real-size electricity system show the robustness of the proposed methodology.**

***Index Terms*—Electricity market, Cournot game theory, Possibility Theory, Chance constraints.**

I. Introduction

Nowadays, most electricity oligopoly markets are regulated on the base of competition among the companies or agents, seeking, from the point of view of the regulators, to establish an auto-regulated price fixing mechanism. In this context, one important challenge for researches has been to obtain proper market behavior models to compute the agent energy productions, using among others approaches, the so-called *Cournot equilibrium* approach (see [1] for a review of its application in the electricity market). The Cournot productions $\{P_e,\ e=1,..,E\}$ of each producer (i.e., the strategies) result from maximizing the individual agent profit $B_e(P_e)=\lambda\cdot P_e-C_e(P_e)$ when the productions of the others agents are supposed to be fixed (λ is the market price and $C_e(P_e)$ the generation costs) and when they do not respond to changes in the market price (i.e., $P_e(\lambda)=P_e$). If the demand curve D is a linear function of λ, the Cournot equilibrium conditions are formulated as:

$$\left\{\begin{array}{l} \lambda = \dfrac{\partial C_e}{\partial P_e} + \mu \cdot P_e \quad \forall e = 1,.., E \\ \lambda = \mu \cdot \left(d - \sum_{e=1}^{E} P_e \right) \end{array}\right\} \tag{1}$$

where the demand at price λ^0 is D^0 and the slope demand is $1/\mu$, i.e., $D=d-(1/\mu)\cdot\lambda$ with $d=D^0+(1/\mu)\cdot\lambda^0$. Note that in the equilibrium the balance equation (demand D equal to the total generation $P_1+..+P_E$) must also be satisfied.

Nevertheless, Cournot equilibrium presents an important weakness due to its high sensibility with respect to the residual demand curves (R.D.C.) of each agent (see [2]). If they are supposed to be linear, then it is (see [3]):

$$\lambda = \mu \cdot \left(d - P_e - P^0_{E-\{e\}}\right) \quad \forall e = 1,.., E \tag{2}$$

where λ is the market price if the considered agent sells the amount of production P_e, and if $P^0_{E-\{e\}}$ is sold by the other agents.

In the literature, the market models found that take into account the R.D.C. sensitivity are not formulated as equilibrium problems, and assume that a probabilistic estimation for these curves is always available (see [1] for a review). Nevertheless, some drawbacks can lead to inappropriate uses of this type of uncertainty modeling since: 1) very often different probability distributions can be fitted for the same set of statistical information (see [4]); 2) sufficient historical information may not be available (see [5]); 3) to obtain a satisfactory computational efficiency with probabilistic models, several simplifying hypotheses are needed (normality, independence) reducing the rigor of the probabilistic approach; and 4) it is very convenient to represent the subjective (usually linguistic) information provided by the experts about the R.D.C. slope behavior. In these cases *Possibility theory* (see [6]) emerges as an alternative tool to model not only the uncertainty but also the imprecision and vagueness (see [7]), and therefore it can be more flexible than probability theory, although, to some extent, less informative (see [8]). That is why in this paper *possibility distributions* have been chosen to model the uncertainty in the R.D.C. slope, which according to (2) coincides for each agent with μ (from now on denoted by $\tilde{\mu}$), which is the inverse of the demand slope. A common point (P_e^0,λ^0) to all possible R.D.C. of each agent is also assumed (see Fig. 1), which could be established by considering an approximated equilibrium solution.

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Additionally, when modeling real electricity markets, it becomes convenient to obtain sensible energy agent productions when they face up different unfavorable, but possible to a certain degree, R.D.C. slope scenarios. Otherwise, seemingly good strategies could lead to a general loss of producers profit, and to a general inefficiency of the power system, that can be avoided by seeking for robust equilibrium solutions. In this paper robust market equilibriums have been computed using two dual approaches introduced in [9], developed for possibilistic objective optimization problems (see for a review [10]).

Next section describes the fundamental theoretical aspects of Possibility Theory to model the uncertainty of the R.D.C. slope. Applying the two dual approaches introduced in [9], section III proposes two alternative and complementary methods to compute a robust Cournot equilibrium. Both approaches have also been combined into a bi-objective linear fractional optimization problem that provides better compromise solutions. Section III.C presents an application to a real power system with a medium term horizon, and discusses the numerical results. Finally, the conclusion remarks are given.

II. Preliminaries

Let Ω be the variability range of the R.D.C. slope $\tilde{\mu}$, and let Π: $2^{\Omega}:\rightarrow[0,1]$ be the *possibility measure* defined on the power set of Ω, with $\Pi(A)$ being the degree of possibility that A occurs. The fundamental axioms of Π are (see [6]):

$$\Pi(\Phi)=0, \Pi(A\cup B)=\max\left(\Pi(A),\Pi(B)\right) \quad \forall A,B\subseteq\Omega \tag{1}$$

A *possibility distribution* π: $\Omega\rightarrow[0,1]$ can be defined from Π as $\pi(\mu)=\Pi(\{\mu\})$ $\forall\mu\in\Omega$, such that $\pi(\mu)$ quantifies the possibility that the variable $\tilde{\mu}$ takes the value μ. In this paper only continuous possibility distributions are considered. In particular LR-possibility distributions (see Fig.2 and [11]) $\pi\equiv\tilde{\mu}=(\mu^L,\mu^R,\alpha\mu^L,\alpha\mu^R)_{LR}$ are used. Functions L,R: $\Re\rightarrow[0,1]$ are differentiable and strictly decreasing in (0,1), L(s)=R(s)=1 for s≤0 and L(s)=R(s)=0 for s≥1.

The principal advantage of LR possibility distributions is that their shape is preserved for many operations performed via the *Extension Principle* (EP) (see [12] and [13]). For example the addition and the multiplication by a scalar are defined by:

$$\begin{gathered}(m^L,m^R,\alpha^L,\alpha^R)_{LR}+(n^L,n^R,\beta^L,\beta^R)_{LR}=(m^L+n^L,m^R+n^R,\alpha^L+\beta^L,\alpha^R+\beta^R)_{LR}\\ x\cdot(m^L,m^R,\alpha^L,\alpha^R)_{LR}=(x\cdot m^L,x\cdot m^R,x\cdot\alpha^L,x\cdot\alpha^R)_{LR} \qquad x\geq0\\ x\cdot(m^L,m^R,\alpha^L,\alpha^R)_{LR}=(x\cdot m^R,x\cdot m^L,-x\cdot\alpha^R,-x\cdot\alpha^L)_{RL} \qquad x<0\end{gathered} \tag{4}$$

III. Some approaches to obtain robust Cournot equilibriums

A robust Cournot equilibrium can be interpreted as a low risk equilibrium, in the sense that it should be somehow insensible to unexpected but possible inputs. Let's analyze the influence of the R.D.C. slope variability into the equilibrium solution:

--When the equilibrium is solved for a given R.D.C. slope μ, but the real R.D.C. slope μ^r is lesser than expected, then according to the equilibrium constraints (see (1)), if the agents keep the same productions P_e, the following results are obtained (see Fig. 3): 1) a lesser market price (λ^r instead of λ); 2) a lesser profit agent ($\lambda^r\cdot P_e-C_e(P_e)$ instead of $\lambda\cdot P_e-C_e(P_e)$); and 3) a loss of the demand utility due to a larger amount of not satisfied demand (D instead of D^r).

--On the other hand, when $\mu\leq\mu^r$ the opposite results are obtained.

In this sense, sensible agents should protect themselves against the first case, trying to reduce the risk of unexpected low profits. This suggests more sophisticated Cournot equilibriums, where market agents take risk minimization into account. In the sequel three different approaches to solve this type of equilibrium are presented. For each solution, the possibility distribution of the market price λ (denoted $\tilde{\lambda}$) can be computed through applying (4) in the following fuzzy equation:

$$\tilde{\lambda}=\tilde{\mu}\cdot\left(d-\sum_{e=1}^{E}P_e\right) \tag{5}$$

A. The primal approach

In this approach each agent look for a set of productions that maximizes a pessimistic (minimum) profit $B_{e,\min}(P_e,\alpha^0_{\max})$, such that the possibility $\prod$ of having profits lesser than $B_{e,\min}(P_e,\alpha^0_{\max})$ is equal to a same value $\alpha^0_{\max}\in[0,1]$ for any agent, that is:

$$\begin{aligned}&\underset{P_e}{Max}\left\{B_{e,\min}\left(P_e,\alpha^0_{\max}\right)=\underset{B_e\left(P_e,\alpha^0_{\max}\right)}{Min}B_e\left(P_e,\alpha^0_{\max}\right)\right\} \quad \forall e=1,..,E\\ &s.t. \qquad \Pi\left\{\tilde{B}(P_e)\leq B_e\left(P_e,\alpha^0_{\max}\right)\right\}=\alpha^0_{\max}\end{aligned} \tag{6}$$

where the fuzzy firm profit in terms of $\tilde{\mu}$ is (D is a function of P_e through the balance equation):

$$\tilde{B}_e(P_e)=\tilde{\mu}\cdot P_e\cdot(d-D(P_e))-C_e(P_e) \quad \forall e=1,..,E \tag{7}$$

Applying (4) and assuming $d\geq D$ (for example when price λ^0 is near to zero) each firm profit has been modeled by a known LR possibility distribution given by (if $d<D$ a different LR possibility distribution is obtained, and the best solution found assuming $d\geq D$ and $d<D$ should be chosen):

$$\begin{gathered}\tilde{B}_e(P_e)=(B^L_e(P_e),B^R_e(P_e),\alpha B^L_e(P_e),\alpha B^R_e(P_e))_{LR} \quad \forall e=1,..,E\\ \left.\begin{cases}B^H_e(P_e)=\mu^H\cdot P_e\cdot(d-D)-C_e(P_e)\\ \alpha B^H_e(P_e)=\alpha\mu^H\cdot P_e\cdot(d-D)\end{cases}\right\}\forall H\in\{L,R\}\end{gathered} \tag{8}$$

According to [9], $B_{e,\min}(P_e,\alpha^0_{\max})$ is reached for $\mu=\mu^L-L^{-1}(\alpha^0_{\max})\cdot\alpha\mu^L$, and therefore problem (6) is equivalent to:

$$\underset{P_e}{Max}\left\{B_{e,\min}\left(P_e,\alpha^0_{\max}\right)=B_e^L\left(P_e\right)-L^{-1}\left(\alpha^0_{\max}\right)\cdot\alpha B_e^L\left(P_e\right)\right\}\quad\forall e=1,..,E \quad (9)$$

This means that the original fuzzy model (6) is simplified into a crisp one that can be solved as in [14], by minimizing the following cost, in the sequel denoted $C_{\max}(\{P_e\},D,\alpha^0_{\max})$ (note that the balance equation is denoted by F):

$$C^0_{\max}=\underset{(P_1,..P_E,D)\in F}{Min}\left\{\sum_{e=1}^{E}C_e(P_e)+\frac{\left(\mu^L-L^{-1}\left(\alpha^0_{\max}\right)\cdot\alpha\mu^L\right)}{2}\cdot\left(\sum_{e=1}^{E}P_e^2+D^2-2\cdot D\cdot d\right)\right\} \quad (10)$$

To solve (10), a resolution procedure similar to the approach showed in [14] could be used.

B. The dual approach

An alternative approach consists in looking for a set of productions that minimizes, for each agent, the possibility of having profits lesser than a minimum profit target $B^0_{e,\min}$, maintaining fixed the productions of the other agents.

$$\underset{P_e}{Min}\left\{\alpha_{e,\max}\left(P_e,B^0_{e,\min}\right)=\Pi\left\{\tilde{B}_e\left(P_e\right)\leq B^0_{e,\min}\right\}\right\}\quad\forall e=1,..,E \quad (11)$$

Asymmetric solutions could be obtained using different $\alpha_{e,\max}$ for the agents, reflecting the existence of agents with different attitude against risk, although a same risk attitude has been assumed ($\alpha_{e,\max}=\alpha$, $\forall e=1,..,E$). According to [9] problem (11) is then equivalent to:

$$\underset{P_e}{Min}\left\{\alpha\left(P_e,B^0_{e,\min}\right)=L\left(\frac{B_e^L\left(P_e\right)-B^0_{e,\min}}{\alpha B_e^L\left(P_e\right)}\right)\right\}\quad\forall e=1,..,E \quad (12)$$

which is equal (see appendix) to minimize the following possibility degree, denoted $\alpha_{\max}(\{P_e\}, D, C^0_{\max})$:

$$\alpha^0_{\max}=\underset{(P_1,..P_E,D)\in F}{Min}\quad L\left(\frac{\sum_{e=1}^{E}C_e(P_e)+\frac{\mu^L}{2}\cdot\left(\sum_{e=1}^{E}P_e^2+D^2-2\cdot D\cdot d\right)-C^0_{\max}}{\frac{\alpha\mu^L}{2}\cdot\left(\sum_{e=1}^{E}P_e^2+D^2-2\cdot D\cdot d\right)}\right) \quad (13)$$

where, if $P^0_{e,\min}$ and $\mu^0_{\min}$ are the productions and the R.D.C. slope respectively giving the profits $B^0_{e,\min}$, then $C^0_{\max}$ is:

$$C^0_{\max}=\sum_{e=1}^{E}C_e(P^0_{e,\min})+\frac{\mu^0_{\min}}{2}\cdot\left(\sum_{e=1}^{E}\left(P^0_{e,\min}\right)^2+\left(\sum_{e=1}^{E}P^0_{e,\min}\right)^2-2\cdot\left(\sum_{e=1}^{E}P^0_{e,\min}\right)\cdot d\right) \quad (14)$$

To solve (13), a resolution procedure similar to the approach showed in the following subsection could be used.

C. The combined approach

A more sensible approach to robust Cournot equilibrium is to determine a compromise solution between primal and dual approaches, maximizing simultaneously the profit $B_{e,\min}(P_e, \alpha^0_{\max})$ and the necessity $\beta(P_e, B^0_{e,\min})=1-\alpha(P_e, B^0_{e,\min})$:

$$\underset{P_e}{Max}\left\{B_{e,\min}\left(P_e,\alpha^0_{\max}\right),\beta\left(P_e,B^0_{e,\min}\right)\right\}\quad\forall e=1,..,E \quad (15)$$

That is, the following bi-objective optimization model (obtained by combining (10) and (13)) must be solved:

$$\underset{(P_1,..,P_E,D)\in F}{Min}\left(C_{\max}\left(\{P_e\},D,\alpha^0_{\max}\right),\alpha_{\max}\left(\{P_e\},D,C^0_{\max}\right)\right) \quad (16)$$

In order to compute a compromise solution it is interesting to define a preference function to prioritize objectives (see [15]). To be able to compare both objectives in a same [0,1]-scale, the following linear functions are proposed:

$$\begin{aligned}p\left(\{P_e\},D\right)&=Max\left(0,\frac{Cu^0_{\max}-C_{\max}\left(\{P_e\},D,\alpha^0_{\max}\right)}{Cu^0_{\max}-Cl^0_{\max}}\right)\\ d\left(\{P_e\},D\right)&=Max\left(0,\frac{\alpha u^0_{\max}-\alpha_{\max}\left(\{P_e\},D,C^0_{\max}\right)}{\alpha u^0_{\max}-\alpha l^0_{\max}}\right)\end{aligned} \quad (17)$$

where the parameters $Cu^0_{\max}$ and $\alpha u^0_{\max}$ are some undesirable objective values for (10) and (13) respectively.

Then, to calculate a compromise solution of problem (16), the minimum operator function has been chosen as preference function to combine $p()$ and $d()$ (see [16] for other compensatory operators):

$$\underset{(P_1,..,P_E,D)\in F}{Max}\ Min\left(p\left(\{P_e\},D\right),d\left(\{P_e\},D\right)\right) \quad (18)$$

which is equivalent to:

$$\begin{aligned}&\underset{(P_1,..,P_E,D)\in F}{Max}\ \theta\\ &s.t.\quad \theta\leq p\left(\{P_e\},D\right)\\ &\qquad\ \ \theta\leq d\left(\{P_e\},D\right)\end{aligned} \quad (19)$$

where, for each set of productions, the auxiliary variable θ can be interpreted as the minimum "distance" to the primal and dual optimal solutions.

This new problem is a nonlinear programming one. To apply linear optimization software, first of all the quadratic functions in (10) and (13) must be linearized, using for example the same piecewise approximation found in [14]. Then, the new problem is linear in P_e but not in θ, and therefore the robust Cournot equilibrium is the solution of the linear feasible region obtained when the higher value of θ is fixed. Thus, this model can be solved using the iterative algorithm suggested in [17].

TABLE I
MOST POSSIBLE R.D.C. SLOPE

Load levels	Peak	Off Peak 1	Off Peak 2
μ^L (c$/KWh/GW)	1/2	1/7	1/9

TABLE II
PARAMETERS OF $p()$ AND $d()$ FUNCTIONS

$Cl^0_{\max}$ (K$)	$Cu^0_{\max}$ (K$)	$\alpha l^0_{\max}$ (pu)	$\alpha u^0_{\max}$ (pu)
$-33\cdot 10^5$	$-25\cdot 10^5$	0.22	1

IV. CASE STUDY

This case study has been implemented in GAMS language (see [18]) and solved using CPLEX solver (see [19]). It has been executed in a 1.8 GHz PC with 524 MB and Pentium IV processor.

A. Case description

The represented model corresponds to the Spanish hydrothermal electric power system. A multi-period case is considered, which has 12 periods (January-December), 2 sub-periods for each period (working days and weekends) and 3 load levels for each sub-period (peak, off peak 1 and off peak 2). There are 7 agents in the market that own 29 hydraulic groups and 101 thermal ones. Together with the balance equation, some additional constraints have been incorporated in the model as for example the technical constraints of each generation group. The resulting case study has then approximately 84.000 constraints and 120.000 variables.

The considered fuzzy R.D.C. slope is modeled by a LR possibility distribution for each load level with $L(s)$=R(s)=$max(1-s,0)$. The width $\alpha\mu^L=\alpha\mu^R$ has been chosen equal to 40% of $\mu^L=\mu^R$ (see Table I).

The risk parameter $\alpha^0_{\max}$ has been fixed in 0.4 pu, and the maximum cost $C^0_{\max}$ in $-3\cdot 10^6$ K$ through previously solving the crisp model of [14] for $\mu=\mu^L-\alpha\mu^L$ and $\mu=\mu^L$, to obtain an estimation of the smaller and larger cost $C^0_{\max}$ respectively. Finally, the parameters of function $p()$ and $d()$ in (17) are showed in table II.

B. Results discussion

Two different types of equilibriums have been compared. The first one, denoted by $\{Pm_e\}$, has been computed for the most possible R.D.C. slope μ^L, and solved with the crisp model presented in [14]. Execution time in this case is about 3 minutes. The second one, denoted by $\{Pr_e\}$, has been obtained with the combined approach proposed in this paper. Execution time has been about 10 minutes, larger than the non-fuzzy approach because of showing stronger non-linear conditions. The maximum satisfaction level θ^* obtained has been 0.95.

To check the robustness of these two equilibriums defined by their solutions $\{Pm_e\}$ and $\{Pr_e\}$, the crisp cost function of [14] that measures the system efficiency can be evaluated under different possible R.D.C. scenarios. Lesser R.D.C. slope scenarios than the most possible one (corresponding to μ^L) can be represented by the parametric function $\mu^{(s)}=\mu^L-s\cdot\alpha\mu^L$ with $s\in[0,1]$. For each scenario $\mu^{(s)}$ the best alternative between $\{Pm_e\}$ and $\{Pr_e\}$ is the one that provides the minimum system cost. By computing $Dif^{(s)}=C(\{Pm_e\},\mu^{(s)})-C(\{Pr_e\},\mu^{(s)})$, it is possible to determine which one is the best alternative, depending on the value of s.

As Fig. 4 shows, in most cases the cost obtained with $\{Pr_e\}$ is lesser than with $\{Pm_e\}$, meaning that $\{Pr_e\}$ is low risk equilibrium in the sense defined in the previous section. Nevertheless, the cost obtained with $\{Pm_e\}$ in the most possible scenario (s=1) is necessarily lesser than with $\{Pr_e\}$, obeying to the fact that risk protection always implies a cost increment.

When market prices are compared for each load level, again in most cases $\{Pr_e\}$ leads to lesser prices than $\{Pm_e\}$, corresponding to larger system efficiency. Fig. 5 shows the fuzzy average market price when the robust equilibrium $\{Pr_e\}$ is considered.

C. Conclusions.

This paper shows that Possibility Theory can be effectively used to model the uncertainty of the residual demand curve (R.D.C.) for an electricity market, where each agent maximizes its benefit based on the Cournot equilibrium conjecture. Two dual criteria have been proposed in order to obtain a robust Cournot equilibrium, protecting the agents against an overestimated R.D.C. slope, which could lead to a significant lost of profits and to a general inefficiency of the power system. These two criteria, dual in the sense showed in [9], have been combined to obtain a better compromise solution, leading to a non-linear programming model that can easily be solved with an iterative algorithm.

The combined approach has been applied to a real large-scale electricity market. It has been checked that the solution it provides is more robust than the one computed for the most possible R.D.C.

Future works could extend the proposed approach to other power related market for which the Cournot framework is not appropriate, like for example markets where agents offer a supply function (or a quantity-price function). Other interesting improvements could be the inclusion of fuzzy constraints, such as those modeling generation units availability, or the power network physical constraints.

APPENDIX

This section proofs the equivalence between problems (12) and (13) as stated in the dual approach.

--On the one hand, the defined robust Cournot equilibrium is obtained solving (12), that is, differentiating $\alpha(P_e, B_{e,\min})$ with respect to P_e and zeroing for all e:

$$\frac{\partial\alpha\left(P_e,B^0_{e,\min}\right)}{\partial P_e}=\left.\frac{\partial L(y)}{\partial y}\right|_{y=L^{-1}\left(\alpha\left(P_e,B^0_{e,\min}\right)\right)}\cdot\frac{\partial\left(L^{-1}\left(\alpha\left(P_e,B^0_{e,\min}\right)\right)\right)}{\partial P_e}=0 \quad (20)$$

As the derivate of $L()$ is strictly negative in the range (0,1) (because is strictly decreasing), then it is:

$$\frac{\partial\left(L^{-1}\left(\alpha\left(P_e, B^0_{e,\min}\right)\right)\right)}{\partial P_e} = 0 \quad \forall e = 1,..,E \tag{21}$$

It is not difficult to show that the above equations are equivalent to the following ones:

$$\frac{\partial B^L_e(P_e) \big/ \partial P_e}{\partial \alpha B^L_e(P_e) \big/ \partial P_e} = L^{-1}\left(\alpha\left(P_e, B^0_{e,\min}\right)\right) \quad \forall e = 1,..,E \tag{22}$$

--On the other hand, the Lagrange function of (13) is:

$$\Lambda\left(\vec{P}, D, \eta\right) = \alpha_{\max}\left(\vec{P}, D, C^0_{\max}\right) - \eta\left(D - \sum_{e=1}^{E} P_e\right) \tag{23}$$

Since in (20), it is immediate to check that:

$$\frac{\partial \Lambda}{\partial P_e} = 0 \Rightarrow \frac{\partial C^L\left(\vec{P}, D\right) \big/ \partial P_e}{\partial \alpha C^L\left(\vec{P}, D\right) \big/ \partial P_e} = L^{-1}\left(\alpha_{\max}\left(\vec{P}, D, C^0_{\max}\right)\right) \quad \forall e = 1,..,E \tag{24}$$

As the lector can check, the left sides of (22) and (24) are equal, that is:

$$\frac{\partial B^L_e(P_e) \big/ \partial P_e}{\partial \alpha B^L_e(P_e) \big/ \partial P_e} = \frac{\partial C^L\left(\vec{P}, D\right) \big/ \partial P_e}{\partial \alpha C^L\left(\vec{P}, D\right) \big/ \partial P_e} \quad \forall e = 1,..,E \tag{25}$$

Besides, according to the definition of $C^0_{\max}$:

$$\tilde{C}\left(\vec{P}, D\right) = C^0_{\max} \Leftrightarrow \tilde{B}_e(P_e) = B^0_{e,\min} \ \forall e = 1,..,E \tag{26}$$

Then, in terms of possibility measure it is hold true:

$$\begin{aligned} &\alpha\left(\vec{P}, D, C^0_{\max}\right) = \Pi\left\{\tilde{C}\left(\vec{P}, D\right) = C^0_{\max}\right\} = \Pi\left\{\tilde{B}_e(P_e) = B^0_{e,\min} \ \forall e = 1,..,E\right\} \\ &\leq \\ &\underset{e=1,..,E}{Min}\, \Pi\left\{\tilde{B}_e(P_e) = B^0_{e,\min}\right\} = \Pi\left\{\tilde{B}_e(P_e) = B^0_{e,\min}\right\} = \alpha\left(P_e, B^0_{e,\min}\right) \end{aligned} \tag{27}$$

Due to the agents independently choose its minimum production level $P^0_{e,\min}$, the equality in the above equation is obtained (see [20]). Therefore (22) and (24) are equal, and the equivalence between problems (12) and (13) is proved.

Note that $\partial\Lambda/\partial D$=0 defines the Lagrange multiplier η for each feasible solution, and $\partial\Lambda/\partial\eta$=0 is the demand balance equation.

ACKNOWLEDGMENT

The authors would like to thanks colleges J. Reneses, E. Centeno and F. J. Santos by the development of the programmed model for the non-fuzzy case and gratefully acknowledge by comments led to improve this paper.

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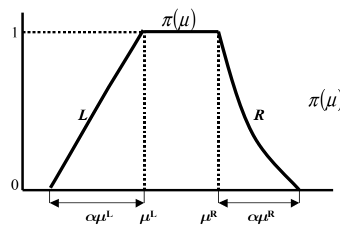


$$\pi(\mu) = \begin{cases} L((\mu^L - \mu)/\alpha\mu^L), & \mu^L - \alpha\mu^L \leq \mu \leq \mu^L \\ 1, & \mu^L \leq \mu \leq \mu^R \\ R((\mu - \mu^R)/\alpha\mu^R), & \mu^R \leq \mu \leq \mu^R + \alpha\mu^R \end{cases}$$

Fig. 2. LR possibility distribution.

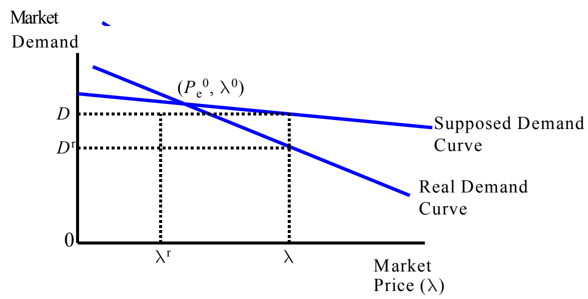
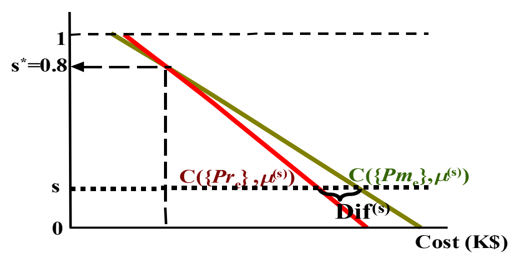


Fig. 3. Demand with $\mu^r < \mu$.



$$Cost \equiv C\left(\{P_e\}, \mu^{(s)}\right) = \sum_{e=1}^{E} C_e(P_e) + \frac{\mu^{(s)}}{2} \cdot S\left(\{P_e\}, \sum_{e=1}^{E} P_e\right)$$

$$Dif^{(s)} = -2 \cdot 10^4 + 9.7 \cdot 10^4 \cdot (1 - s) \begin{cases} < 0 & if \quad s > s^* = 0.8 \\ \geq 0 & otherwise \end{cases}$$

Fig. 4. Results Discussion.

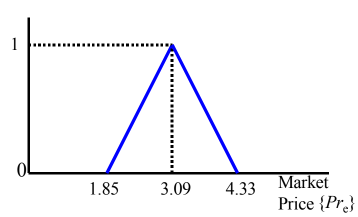


Fig. 5. Average market price possibility distribution.

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