Energy auctions for regulated demand in the Iberian market: a proposal

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***Keywords*—MIBEL, regulated demand, auction.**

***Abstract*-- The possible organization of auctions to meet regulated demand in the future Iberian electricity market (MIBEL) has prompted the present study. The current structure and operation of such auctions are reviewed and a procedure suitable for their institution on the MIBEL is proposed, in the context of the limited transmission capacity existing between Spain and Portugal.**

I. INTRODUCTION

ENERGY auctions constitute one of the fundamental components of electricity market operation in general and the Spanish peninsular market in particular. Nonetheless, the use of centralized medium-term (from a few months to several years) auction mechanisms to cover regulated demand is much less prevalent.

While electric power markets are organized very differently in the various systems where they are in place, in most the products traded typically cover a wide range of time horizons. In this regard, the forward contracts traded on the Portuguese electricity pool (OMIP) are particularly relevant to the medium term (to which the present proposal is primarily geared), although trading volumes are small. In the context of the European Union, forward contracts are known to be traded on organized markets (Powernext, EEX, AEX, Belpex, Nordpool, ...) or over the counter, or to take contracts traded on those markets as a reference. Similar instruments are in place in the rest of the world as well. And yet these markets have been shown to be inappropriate or insufficient to solve certain problems that arise on a regular basis, such as supply for regulated customers or customers accorded special rates for social or other reasons, or the effective control of the possible exercise of market power.

In the framework of the European Union, the mechanisms that most closely resemble these auctions are the virtual power purchases or VPPs that have been instituted in several Member States, including France , Belgium , Denmark and, recently, Spain. In such auctions the largest operator or operators auction part of their generating capacity to smaller scale agents, a measure intended to raise liquidity and competitive pressure on the market for electric power.

The use of auctions to cover regulated demand has been discussed less profusely in Europe. Outside the continent, two systems have accumulated experience of value in this context:

- In Brazil distributors and large-scale consumers must conclude contracts with generators to cover their expected demand. Regulations distinguish between liberalized demand (typically large-scale consumers) and captive consumers. The five-year demand associated with the latter is aggregated by the Government, which offers generators supply contracts (Contratos de Comercializacão de Energia no Ambiente Regulado). Contracts are allocated based on the outcome of a non-standard auction [1].
- In the United States, the public utility commissions of various of the PJM electric power market states have organized auctions to provision their regulated demand (specifically, auctions have been held in Illinois, Maryland, Delaware and New Jersey). The characteristics vary from one state to another, but in all cases the mechanism is a descending auction and the term is one year. The rate paid by regulated customers is set on the grounds of the results of these auctions.

The research reported in the present article was occasioned by the introduction in Spain of auctions to cover regulated demand, which are governed by a recent ministerial order, ITC/400/2007, of 26 February 2007. Although the order has not yet been fully implemented (e.g., the order provides for two products, "base load" and "modulated load", while auctions have been held for the former only), it establishes the Spanish legal framework on the subject and appears to have served as well as a guide for the Portuguese regulator.

The remainder of this article is organized as follows. The section below discusses some of the relevant theoretical questions relating to auction organization. Section 3 provides a critical description of the auctions for the purchase of power to cover regulated demand held in Spain through November 2007. Section 4 analyzes the auction mechanism put forward in [2], which is the basis for the mechanism proposed in section 5 of this article for the MIBEL.

II. THEORETICAL CONSIDERATIONS

Auction theory, until recently a minor and relatively unknown branch of economics, is presently one of the areas of the discipline most actively researched [3,4]. One of the primary reasons for this change is the deregulation of network industries, telecommunications and energy in particular . The auctions involved are:

- Of multiple units. In "classic" auctions only one item (a painting, for instance) is up for sale. In electricity auctions, even where a single product is auctioned (base load, for example), it is bundled into multiple packages (the electric power required over a three-month period, for instance, divided into 50-MW brackets).
- inter-related. For example, the cost for a generator supplying certain amounts of base and modulated load is

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not the simple sum of the base and modulated load functions, but a joint function for both variables that cannot, in principle, be readily broken down due to the nature of the operating constraints that electricity generation entails.

The use of auctions for this type of problems is very recent (the case of the American communications spectrum was very likely the first example [5]). For that reason, theoretical fundamentals on which to base a satisfactory analysis of the problem have yet to be fully developed. Most of the results provided by classical auction theory are difficult to extrapolate to this context and no systematic approach has been put forward to deal with the risk factors, other than the ones traditionally considered, associated with such auctions.

In any event, results have been reported that prove that certain auction procedures can ensure efficient and transparent allocation when the goods auctioned are substitutes. This is the case of auctions for distributing electric power, in which the base load can be regarded to be a substitute for peak load, or where products with different delivery periods are substitutes [6].

Nonetheless, any attempt to extend these mechanisms to the Iberian market would raise problems that would have to be analyzed in detail. The reason is the limited inter-connection capacity existing between Spain and Portugal. Two possibilities may initially be considered:

1. To completely ignore the transmission constraints in the auction. In this case the parties to the bilateral agreements stemming from the auction would have to assume the risk of securing the transmission capacity required for delivery (in the spirit of the Order, that risk might be assumed by the seller). Such additional risk would indisputably raise bid prices, affecting the outcome of the auction. Under the specific conditions prevailing in the Iberian market, the increase involved might well be significant.
2. To take the limits to transmission into consideration in the auction, either implicitly in the clearing mechanism or explicitly as an additional product (transmission capacity). Either hypothesis entails the appearance of supplementary products. Auction theory provides less satisfactory solutions in this case, which could, however, be sought with designs resembling a "standard" descending auction.

Such problems are partially present even in auctions involving the Spanish system only, due to the possible participation of cross-border agents. Given the relatively small impact of the energy coming from France, however, and the agreements presently in effect, such problems are unlikely to be of much practical importance.

The design for a mechanism put forward in this article adopts this second approach, in which transmission limitations are taken into consideration.

III. Possible General Framework of Auctions for Covering Regulated Demand in the MIBEL

The establishment of joint auctions for covering regulated demand on the MIBEL will naturally be subject to an agreement between the Spanish and Portuguese regulators, which has yet to be forthcoming. Indeed, talks have not even been formally initiated. Moreover, the auctions presently held in Spain and Portugal both may be expected to condition possible future joint initiatives. In this regard, an analysis of the legislation on these auctions, specifically Ministerial Order ICT/400/2007 of 26 February 2007, should yield some relevant background information.

A. Auction structure

The Order provides that:

The energy to be supplied by each seller will be allocated by means of an auction procedure in which, after the amount of energy to be supplied for all the distributors defined in Article 2 is established, along with the opening price, the latter is progressively lowered until supply matches demand.

In other words, the Order provides for a descending auction, as usual in electricity markets [7]. The Annex to the Order lays down certain additional requirements, such as:

- Suitable competitive pressure *(If the number of eligible subjects is regarded to be insufficient or where there is reason to believe that competitive pressure is insufficient, the body responsible will suspend the auction after serving notice upon the National Energy Commission to that effect).*
- Clarification of descending price *(The opening price for the auction will be set in a manner such that it is higher than the anticipated closing price,... From one round to the next, prices may not be raised for any product whatsoever).*
- Limits to the information provided to agents during the auction *(The exact amount of excess supply for each product in each round will not be disclosed).*
- Criteria for lowering prices between successive rounds *(The formula for lowering prices for each product between rounds will be based on the excess supply for each product. The formula, which will not be disclosed. may contain a random or discretional component).*
- Simultaneous auctioning of different products, simultaneous auction closing and need to restrict the changes in agents' bids for the different products to guarantee convergence in the procedure. *(Auctions for products having the same duration will be held simultaneously, and agents will be allowed to transfer their bids from one product to another between rounds. Such transfers will be limited... A subsequent provision states that Auctions will be closed for all products at the same time, when the supply of all the products auctioned has been depleted).*

The Order includes certain additional clarifications and recommendations. Generally speaking, the proposal is similar to the one implemented in some of the PJM market states. Consequently, there is a wealth of publicly available information and accumulated experience that might be profitably analyzed by the body responsible for the auction, the competent authorities and the agents concerned .

Several remarks on these provisions follow:

- The existence of sufficient competitive pressure is, indeed, of vital importance. It can be evaluated in terms of a number of magnitudes, the most prominent being the number of agents submitting bids and the maximum amount of power they are prepared to tender. Auction rules might lay down limits for indicators based on these or other magnitudes as requisites for holding such events.

- The Order places limits on the amounts tendered that can be switched from one product to another. The rules adopted to this end must be suited to the aim pursued and should not be unnecessarily complex.
- No provision is made in the Order for reserve prices. Nonetheless, it may be advisable to set such amounts (such as in Brazilian auctions) to lower the risk of collusion and predatory behaviour [8]. In this case, the opening price should be higher, and perhaps clearly higher, than the reserve price in order to afford agents the opportunity to converge on an efficient solution. It is probably also advisable not to disclose the reserve price to prevent the appearance of a "focal point" that would facilitate collusive behaviour. The rules for the auctions held in Spain to date have established a confidential reserve price. An alternative to reserve prices that may be similarly effective in terms of limiting incentives for collusive behaviour would consist in proposing an elastic demand curve [9].Such techniques are particularly fitting for auctions involving overlapping products (the last three months of six-month products auctioned in September overlap with the quarterly products auctioned in December, for instance).
- To ensure the existence of sufficient competitive pressure, the information received by the agents on auction developments must be limited. In the auctions held in Spain, for instance, information on excess supply has been withheld from bidders in the last few iterations.

B. Product definition

The Order includes a definition of the products to be auctioned:

- Base load: the seller commits to supplying a steady flow of a given amount of electric power in every single hour throughout the period defined in the contract. The capacity specified for each seller is determined competitively, by auction.
- Modulated load: the seller commits to supplying a fixed percentage of a flow of power that varies from hour to hour throughout the period defined in the contract. The percentage specified for each seller is determined competitively, by auction.

Essentially, the two products defined are a constant power flow (base load) and power supplied in accordance with a pre-defined profile (modulated load) during the delivery period. Nonetheless, the Order fails to define these products in any detail. Specifically:

- Although the Order provides that the reference amount to be used to define the "modulated load" product must be computed from the best estimate of [the…] load curves corresponding to the period defined for each auction, along with the load curve for the same period in the preceding year, no details are provided on the procedure to be followed or the expected final result. Two extremes could be envisaged: in one, the final product would be a profile that would differ from hour to hour throughout the period, and in the other, it would consist in a standard profile that would vary weekly, establishing a peak and a valley value, and the hours defined as one or the other. The former case would provide better coverage for regulated demand, while the second would facilitate the definition and sale of products in secondary markets. In the event of auctions for products having a delivery period of over three months, it would also facilitate power purchases to cover needs in auctions held in the interim. In any event, the friction between adaptation and liquidity is present in final product design.
- In principle, the buyer commits to specifying the amounts purchased. This means that on occasion such amounts may either be insufficient to cover regulated demand or exceed actual requirements (due to prediction error, for instance). It may, therefore, be advisable to consider the possibility of making allowance for deviations between actual consumption and the amounts purchased. The design of the respective regulation must take account of short-term market configuration, as well as the characteristics of, and assets available and regulations applicable to the agents participating in auctions and such markets.

IV. The Auction Mechanism

A. The Ausubel, Cramton and Milgrom hybrid auction

The auction mechanism proposed is based on [2]. These authors proposed a two-step hybrid mechanism, which is reviewed below and adapted as needed for the case addressed in this article. This auction mechanism is geared to the simultaneous purchase of several products (peak and modulated load, for instance).

The first phase of this hybrid mechanism is a descending auction. The auctioneer announces a very high lead price, receives the sums bid by each agent and pools them. Where the aggregate supply exceeds demand the auctioneer proposes a lower price in keeping with the excess. This phase concludes when the aggregate supply for each product is lower than the demand.

The second phase is a proxy auction in which proxy agents send generating bids (pursuant to the cost functions provided by the agents) to the auctioneer, who selects them in accordance with a payment minimization criterion. This phase comes to an end when no additional bids are received.

Ausubel, Cramton and Milgrom base the defence of their mechanism on three main arguments. Perhaps the most relevant to the purposes of the present article is the third: the success of descending auctions in electric power markets.

The rest of this article focuses on the first of the above two phases. That notwithstanding, a few remarks about the possible suitability of a "proxy" phase are included in the conclusions, in section 6 below.

The descending auction proposed by these authors has two characteristics or activity rules that are of cardinal importance to the following discussion. The first is that the bids are firm. In other words, if the auctioneer proposes a price vector p, each agent i proposes a bid vector $x_i(p)$ that remains in effect throughout the auction .

The second condition is a generalization of the activity rule whereby, in single unit auctions, the amount tendered must decline with price. In this type of auctions, the rule (nearly) guarantees that the auction will reach completion: the price is lowered until supply equals demand . The above generalization is based on reasoning that should be reviewed in some detail.

Let $C_i(x_i)$ be the cost to agent i of supplying demand x_i. This cost is not, of course, the cost of the operation only, but includes items such as the opportunity cost of committing the resources needed in this auction instead of in subsequent markets. Take the bids submitted for two prices, p and p'. A sincere agent prefers to produce $x_i(p)$ rather than $x_i(p')$ when the price is p. In other words, the profit (revenue less cost) is greater in the former than in the latter case:

$$p \cdot x_i(p) - C_i(x_i(p)) \geq p \cdot x_i(p') - C_i(x_i(p')) \quad (1)$$

Analogously, the agent prefers to produce xi(p') rather than xi(p) when the price is p'.

$$p' \cdot x_i(p') - C_i(x_i(p')) \geq p' \cdot x_i(p) - C_i(x_i(p)) \quad (2)$$

The difference between the two inequalities is called the "revealed preference activity rule".

$$(p - p') \cdot (x_i(p) - x_i(p')) \geq 0 \quad (3)$$

Where only one product is involved, the rule simply means that the amount tendered rises with price. It is, however, less restrictive than the rule (in place in some auctions) that requires the supply of all products to be monotonic functions of their prices, for it allows for the partial switching of bids from product to product. Nonetheless, as in alternative auction models, such switching is limited by impeding the aggregation of the amounts tendered for different products.

B. Discussion of the Ausubel, Cramton and Milgrom proposal

From the mathematical standpoint, (3) establishes that operator $x_i(p)$ is monotonic, and that consequently the aggregate supply operator $x(p) = \sum_i x_i(p)$ is likewise monotonic. Where a single product is involved, this means that each operator $x_i(p)$ can be computed from be the sub-gradient of a convex "cost function", expressed as $\tilde{C}_i(x_i)$ That is to say,

$$p(x_i) = \frac{\partial \tilde{C}_i(x_i)}{\partial x_i} \quad (4)$$

If all agents' cost functions were in fact convex, in a perfect market, standard microeconomic arguments would lead agents to tender bids equal to their marginal costs, i.e.,

$$C_i(x_i) = \tilde{C}_i(x_i) \quad (5)$$

From the standpoint of auction mechanism design, however, it is the properties of $x_i(p)$ or, equivalently, $\tilde{C}_i(x_i)$ that matter. Since these functions are convex, clearing the market is equivalent to solving

$$\begin{aligned} \min \quad & \sum_i \tilde{C}_i(x_i) \\ \text{s.t.} \quad & \sum_i x_i = d \end{aligned} \quad (6)$$

where d is the total demand for the product.

Problem (6) is a convex problem with a separable objective function. This suggests that techniques such as Lagrangian relaxation should be used. Two fundamental items need to be considered in these algorithms:

1. A price p^k is proposed in every iteration k. The amounts x_i^k would be determined from the sub-gradient of $\tilde{C}_i(x_i)$:

$$x_i^k = \text{sol}\left[\frac{\partial \tilde{C}_i(x_i)}{\partial x_i} = p^k\right] \quad (7)$$

In practical terms, the sub-problems are "solved" by sending proposed prices to the agents who, in turn, return their proposals for amounts. Constraint (3) ensures that, in fact, it as if they were solving a convex optimization sub-problem.

2. If $\sum_i x_i^k$ differs significantly from d, a new price needs to be proposed. The standard rule is to do so according to the value of the duality gap, i.e., according to the rule below:

$$p^{k+1} = p^k - \alpha^k \left(\sum_i x_i^k - d \right) \quad (8)$$

This is the type of rule that is in fact generally used in real auctions. Moreover, guidance on the selection of the most appropriate value for constant α^k can be found in the richly developed optimization theory now in place.

In short, the relationship existing between auction rules and an associated optimization problem provides guidance for formulating such rules.

V. Generalizing the Procedure

Unfortunately, when more than one product is involved, monotony does not suffice to guarantee the existence of a convex function from which bids can be derived. This problem can be solved in two different ways:

1. By replacing activity rule (3) with another rule that guarantees the existence of this function.
2. By seeking inspiration for the development of auction mechanisms not in optimization theory, but in the theory of variational inequalities which incorporates such requirements naturally.

This section will address the second possibility, for rule (3) is regarded to be very natural and a satisfactory solution is considered possible within the framework of variational inequality theory.

The general approach assumes an implicit auction system. In each such system, a series of products are in demand. In some cases, demand may be nil. For example, some products can be defined on the basis of the demand for modulated load to supply Portuguese consumers, which may exhibit a profile that differs substantially from the demand pattern generated by Spanish consumers. Two products could therefore be defined: the Portuguese modulated load delivered in Portugal and the Portuguese modulated load delivered in Spain. Demand for the second product would be nil. The amounts demanded are ranked in a vector d .

Each agent i may submit bids $x_i(p)$ for each product, given a price vector p . Agents may decide not to submit bids for any one or for several of the products, in which case the respective component of vector $x_i(p)$ is 0. In any event, function $x_i(p)$ must be monotonic. In other words, the bids are firm and the revealed preference activity rule shown below must hold:

$$(p-p')\cdot(x_i(p)-x_i(p'))\geq 0 \tag{9}$$

Finally, flows have to be defined for each type of product as the difference between the supply and demand for a given type of product in Portugal and Spain. For instance, the modulated Portuguese load flow is demand for Portuguese modulated load in Spain (possibly nil) less the aggregate supply of the Portuguese modulated load supplied in Spain, less Portuguese modulated load in Portugal, plus the aggregate supply of the Portuguese modulated load supplied in Portugal. Flow vector I can consequently be expressed as

$$I=A\left(\sum_i x_i-d\right) \tag{10}$$

where A is a constant matrix, known in electrical engineering as the node-branch incidence matrix, whose elements are 0, 1 or -1.

Moreover, transmission is subject to limitations due to the finite capacity between Spain and Portugal. Such limitations could be established by means of linear constraints on power flows. Assume, for instance, an auction for monthly products in which one unit of each product is equal to 720 MW-h. Since there are 720 hours in a month, a 720 MW-h base load flow per month is tantamount to a flow of 1 MW per hour across the inter-connection. A flow of 720 MW-h of Spanish modulated load, in turn, may entail a flow of, say, 2 MW at 8:00 p.m. and 0.5 at 4:00 a.m. on week days. These profiles, which are constant given the product definition, can be represented as a vector, f_h, which in principle differs for each hour of the month. If the transmission capacities in each direction (Spain-Portugal and Portugal-Spain) are, respectively, $\underline{I}_h$ and $\overline{I}_h$ for each hour, the following constraint will be in place:

$$\underline{I}_h\leq f_h\cdot I\leq \overline{I}_h \tag{11}$$

In addition, a transmission price can be defined for each product class μ as the difference between the prices for such classes in Spain and Portugal. In other words, the transmission price of Portuguese modulated load is the difference between the price of the Portuguese modulated load in Spain less the price of the Portuguese modulated load in Portugal. Mathematically,

$$\mu=Ap \tag{12}$$

It may be advisable to express μ as the difference between two non-negative amounts:

$$\mu=\underline{\mu}-\overline{\mu},\underline{\mu},\overline{\mu}\geq 0 \tag{13}$$

Therefore

$$\underline{\mu}-\overline{\mu}=Ap \tag{14}$$

The auctioneer's problem can be divided into two parts. On the one hand, he must determine prices p and flows I such that the sum of the aggregate supply in each country plus imports less exports meets the demand for each product:

$$\sum_i x_i-d-A^TI=0 \tag{15}$$

where A^T is the transpose of the incidence matrix.

Furthermore, the transmission price should be nil if transmission capacity is not a limiting factor.

To pose the auction as a variational problem, the excess supply function $e(p)$ is defined as follows:

$$e(p)=\sum_i x_i(p)-d \tag{16}$$

The following conditions must therefore hold:

- Supply-demand equilibrium
  $$e(p)=A^TI \tag{17}$$
- Transmission price as arbitration factor
  $$\underline{\mu}-\overline{\mu}=Ap \tag{18}$$
- Transmission constraints
  $$\underline{I}_h\leq f_h\cdot I\leq \overline{I}_h \tag{19}$$
- If the preceding inequality is not binding,
  $$\underline{\mu}=\overline{\mu}=0 \tag{20}$$

The variable vector, v, is defined as:

$$v=\left[p,\underline{\mu},\overline{\mu},I\right]^T \tag{21}$$

Operator $F(v)$ can be defined as:

$$F(v)=\left[e(p)-A^TI,I,-I,Ap-\underline{\mu}+\overline{\mu}\right]^T \tag{22}$$

Feasibility set T, containing transmission constraints (10) and (12), is in turn defined as:

$$\mathrm{T}=\left\{(p,\underline{\mu},\overline{\mu},I)/\underline{I}_h\leq f_h\cdot I\leq \overline{I}_h,\underline{\mu},\overline{\mu}\geq 0\right\} \tag{23}$$

The variational problem $\mathrm{VI}(\mathrm{T},F)$ is defined as:

Auction problem:

Find $v^*\in\mathrm{T}$ such that $F(v^*)\cdot(v-v^*)\geq 0,\forall v\in\mathrm{T}$

This problem can be solved if constraint set T is convex and operator F is monotonic [10]. T is obviously convex. As far as F is concerned, the revealed preference rule ensures that every $x_i(p)$ is monotonic. Therefore, excess demand $e(p)$ is likewise monotonic. It can therefore be readily seen that

$$
\begin{aligned}
&(F(v)-F(v'))\cdot(v-v')= \\
&(e(p)-e(p'))\cdot(p-p')-(A^T I-A^T I')\cdot(p-p') \\
&+(I-I')\cdot(\underline{\mu}-\underline{\mu}') \\
&-(I-I')\cdot(\bar{\mu}-\bar{\mu}') \\
&+(Ap-Ap')\cdot(I-I')-(\underline{\mu}-\underline{\mu}')\cdot(I-I')+(\bar{\mu}-\bar{\mu}')\cdot(I-I')= \\
&(e(p)-e(p'))\cdot(p-p')\geq 0
\end{aligned}
$$

Furthermore, the price vector is not delimited in the solution, which means that the associated component in F, i.e. $e(p)-A^T I$, should be cancelled. Indeed, if any of the components were positive, a vector v equal to v^*, but with the price corresponding to the non-zero component smaller than p^*, would meet the following condition: $F(v^*)\cdot(v-v^*)<0$. An analogous argument could be put forward if any of the components were negative.

Analogous arguments may be used to analyze transmission prices. If, within T, it is feasible for $I>I^+$, then $\underline{\mu}=0$ for reasons similar to those set out in the preceding paragraph. Conversely, if $I<I^+$ is possible, then $\bar{\mu}=0$. If both are possible, then $\bar{\mu}=\underline{\mu}=0$. In other words, the prices of comparable products tend to converge in the two countries.

Here, as in the case for a single product, variational inequality theory can be invoked at this point to suggest auction mechanisms. One possibility is to apply the Khobotov (or dual projection) algorithm [11] instead of Lagrangian relaxation. Each auction iteration would be as follows:

1. Agents are asked to submit bids $x_i(p^k)$ compliant with the revealed preference rule (equivalently, guaranteeing that operator $x_i(p)$ is monotonic) for a price vector p^k.

2. The following vector is formed: $v^k=\left[p^k,\underline{\mu}^k,\bar{\mu}^k,I^k\right]^T$

3. The excess supply is calculated: $e^k=\sum_i x_i(p^k)-d$

4. The value of F^k is calculated as

$$F^k=\left[e^k-A^T I^k, I^k, -I^k, Ap^k-\underline{\mu}^k+\bar{\mu}^k\right]^T$$

5. The new variable vector is calculated as:

$$\tilde{v}^k=\mathrm{Proj}_{\mathrm{T}}\left[v^k-\alpha^k F^k\right]$$

where $\mathrm{Proj}_{\mathrm{T}}$ is the projection on T and α^k a judiciously chosen constant.

6. Agents are asked to submit bids $x_i(\tilde{p}^k)$ compliant with the revealed preference rule for the new price vector $\tilde{p}^k$.

7. $\tilde{e}^k$ and $\tilde{F}^k$ are calculated.

8. The following are then calculated:

$$v^{k+1}=\mathrm{Proj}_{\mathrm{T}}\left[v^k-\alpha^k\tilde{F}^k\right]$$

9. The results are checked for convergence, and if it is not reached, k is updated to $k+1$ and the procedure is iterated from step one.

Note that as in the case for a single product, price updating is proportional to the excess supply. Initially, the various products may be priced very high and prices $\underline{\mu}$, $\bar{\mu}$ and the values of flows I may be zeroed. Existing theory may provide reasonable values for constants α^k [12], although it may be more difficult to prevent fluctuations here than in the case of a single product (in other words, to prevent the price of some of the products from rising instead of falling in some of the iterations). Nonetheless, the monotony of F guarantees convergence of the algorithm.

VI. CONCLUSION

A mechanism to auction the procurement in the Iberian electricity market has been proposed. The proposed mechanism can be complemented by a proxy phase that can address the non-convex nature of production cost functions. The proposed mechanism can be also readily generalized to networks that can be represented by means of sets of convex constraints, such as the distribution factors or the DC representation commonly used in meshed networks.

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