Electricity forward and volatility curves computation based on Monte Carlo simulation

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***Abstract*-- As a result of the deregulation processes, liberalized markets, where electricity futures and derivatives are traded, have arisen all over the world. Utilities, consumers, traders and, generally, market agents must do quantitative assessments of their positions. Basic analytical data are the forward and volatility curves of the traded products. However, electricity price dynamics is very different of other commodities prices dynamics. Furthermore, electricity prices of different markets are usually very different of each other. As consequence, most analytical approaches to compute forward and volatility curves, as well as other statistics useful to risk management tasks, are very complex or do not exist. In this paper, we propose to compute the forward and volatility curves by Monte Carlo simulation. The main contribution lies in the used variance reduction techniques, needed to achieve this objective at reasonable computational cost. A case example consisting of the study of the EEX prices is also provided.**

***Index Terms*-- Electricity derivatives, Volatility, Control variates, Variance reduction, Monte Carlo.**

I. INTRODUCTION

NOWADAYS there are many power systems whose regulation has changed or is being changed to achieve liberalized markets, with the aim of obtaining a more efficient decision-making process. One of the main result of this deregulation process is that the risk of the activity has been moved to the generating firms, which have not assured their profit any longer. In this new framework, managing risk is a central problem for the firms, and this fact has allowed for future and other derivative trading to become an increasingly used tool (see for instance [1]).

Maybe the most important characteristic of the electricity concerning price evolution is the non-storability. As the consumption pattern in power systems is strongly seasonal, electricity prices will be strongly seasonal as well. Moreover, this characteristic causes that prices temporally in extreme values will tend in a relatively short period to a more stable level, which is called mean reversion. Finally, spot prices are subject to frequent jumps, due to failures in the system, strategic behavior of the firms, or other causes [2]. Furthermore, historical data of electricity markets show that the volatility of the prices is higher during crisis periods than in quiet ones, i.e. price volatility varies over the time, showing mean reversion behavior. This kind of behavior can be modeled by GARCH processes [3], [4].

Most of the derivative pricing theory assumes a price evolution with constant volatility. However, when volatility variation is taken into account, there are no closed expression to valuate electricity derivatives. To cope with this question, this paper proposes a methodology which overcomes the problem by stating the evolution dynamics of forward curves from a general price model, so that it is possible from a set of price scenarios, generated by Monte Carlo simulation, to obtain a set of forward curve scenarios and obtaining from them the volatility curve of a future contract at any date of the time scope.

However, a great number of price scenarios is commonly required to get a precise result. In order to deal with this problem, the model proposed in this paper reduces the number of scenarios required for the estimation by means of control variates and antithetic variates techniques. A specific set of control variates and a non-standard antithetic variates techniques are proposed.

The sequel is organized as follows. Section II provides a general sight of the method. Section III details the variance reduction techniques used for computing statistics. A case study, based on real EEX data is provided in section IV. In section V we state our conclusions. Finally a mathematical proof is developed in the Appendix.

II. GENERAL METHODOLOGY

When valuating most European derivatives, forward and volatility curves are the only knowledge required. The reason is that these contracts can be usually written as derivatives of future contracts (since futures converge to the spot price at the delivery date). On the other hand, most of electricity derivatives are based on the average price of electricity in some specific period (for instance, average price in May). However, even if daily prices were described by log-normal distributions (and indeed they are not, due to GARCH effects) the average of log-normal distributions is not log-normal. Therefore, analytical expressions of the forward and volatility curves do not exist (or if they do exist, they are approximate ones, see for instance [5]). In this section we develop a new methodology to obtain these curves from a set of price scenarios, and consequently a general algorithm to price any European derivative.

Most of the models developed in the literature (including

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the one considered in this paper) can be expressed in the following moving average way :

$$p_T = e^{d_T + y_T}$$
$$y_T = y_0 + \sum_{t=1}^{T} h^A{}_{T-t+1} u_t^A + \sum_{t=1}^{T} h^B{}_{T-t+1} u_t^B + \ldots$$
$$u_t^i \sim N\left(\mu_{i,t}, \sigma_{i,t}\right)$$

where the logarithm of the prices is the sum of a deterministic component d_t, which in general includes the seasonality and trend of the time series, and an stochastic one y_t. The latter is the sum of several moving average processes, with GARCH volatility, expressed by the time-dependent coefficients $h_k^{A,B,\ldots}$ and the innovations $u_k^{A,B,\ldots}$.

However, the above model of prices describes the natural process of prices over the time, not the risk-neutral one. In order to obtain the risk-neutral model, one can simply add a risk aversion factor λ calibrated from the market (see for instance [6]). Hence the model can be rewritten as follows:

$$p_T^N = e^{d_T + y_T^N}$$
$$y_T^N = y_0 + \sum_{t=1}^{T} h^A{}_{T-t+1} \left(u_t^A - \lambda_t^A\right) + \sum_{t=1}^{T} h^B{}_{T-t+1} \left(u_t^B - \lambda_t^B\right) + \ldots$$
$$u_t^i \sim N\left(\mu_{i,t}, \sigma_{i,t}\right)$$

Hereafter we will consider one risk factor, for simplicity, since all the following can be easily generalized to the several risk factors case. So the model is:

$$p_T^N = e^{d_T + y_T^N}$$
$$y_T^N = y_0 + \sum_{t=1}^{T} h_{T-t+1} \left(u_t - \lambda_t\right) \qquad (1)$$
$$u_t \sim N\left(\mu_t, \sigma_t\right)$$

On the other hand, the forward curve is the discounted risk-neutral expectation of the electricity prices for several delivery dates. In this regard, it is necessary to remark that a constant and known interest rate is herein considered (the rational for this assumption is that currently interest rates are low, so fluctuations of the risk-free rate have a low impact as well). So, the forward price at time *t* of 1MWh delivered at time *T* is the expected value:

$$F_{t,T} = \mathbf{E}_t\left[e^{-r(T-t)} p_T\right] \qquad (2)$$

As shown in the appendix A, forward prices must fulfill the following equation:

$$\ln F_{t+1,T} - \ln F_{t,T} = r + h_{T-t+1}\lambda_t - \frac{1}{2} h_{T-t+1}^2 \sigma_t^2 + h_{T-t+1} u_t' \qquad (3)$$
$$u_t \sim N\left(0, \sigma_t\right)$$

The proposed algorithm for valuing European derivatives has three steps:

1. By generating a number of Montecarlo scenarios according (1), the initial forward curve $F_{0,T}$ is obtained by estimating expectation in (2) by the sample mean.
2. By using the same set of Montecarlo scenarios (i.e., innovations u_t), a set of Forward curves $F_{t,T}^r$ (one for each scenario, denoted *r*) is computed according (3).
3. By using this set of forward curves, the value of the required derivatives are computed as the sample statistics.

Variance reduction techniques are used in step 1 (computation of the initial forward curve) as well as in step 3 (computation of derivatives' values).

In order to illustrate basic idea in step 3, let us assume that our portfolio contains a future on the average price of electricity in the period $\left[T_1, T_2\right]$. It is desired to value the volatility (equivalently, the standard deviation) of this contract assuming its sale at a date $T_0 < T_1$. The reason for that could be that T_0 is a mark-to-market date in which it is required to estimate the portfolio risk. In any case, for each scenario *r* it can be written that

$$F^r_{T_0,[T_1,T_2]} = \text{average}\left(F^r_{T_0,T_1}, \ldots, F^r_{T_0,T_1}\right)$$

and the sought standard deviation is the sample value

$$\sigma_{T_0,[T_1,T_2]} = \text{std}\left(F^1_{T_0,[T_1,T_2]}, \ldots, F^R_{T_0,[T_1,T_2]}\right)$$

III. GENERATION AND ANALYSIS OF PRICE SCENARIOS

One of the main problem that the simulation approach faces is that one must be able to generate and analyze a great number of scenarios. In other words, the greater the estimate accuracy sought is, the greater the number of scenarios required. Moreover, it is a well-known facts that Monte Carlo methods accuracy increases with the square root of the number of scenarios. This usually means an infeasible computational effort. To cope with these questions it is necessary to apply variance reduction techniques. Specifically the model proposed in this paper reduces the number of scenarios required for the estimation by means of control variables and antithetic variables techniques. A specific set of control variables and a non-standard antithetic variables techniques are proposed.

The model of prices in [5] can be written to follow the general equation (1), where the standard deviation follows a GARCH process. That is:

$$p_T^N = e^{d_T + y_T^N}$$
$$y_T^N = y_0 + \sum_{t=1}^{T} h_{T-t+1} \left(u_t - \lambda_t\right)$$
$$u_t \sim N\left(\mu_t, \sigma_t\right)$$

where

$$u_t = k_u + \eta_t$$
$$\sigma_t^2 = k_\sigma + \sum_{i=1}^{P} a_i \sigma_{t-i}^2 + \sum_{j=1}^{Q} b_j \eta_{t-j}^2$$
$$\eta_t \sim N\left(0, \sigma_t\right)$$

Although standard, the former model is not the most amenable for numerical computations. However, by introducing a new variable according:

$$\eta_t = \sigma_t \varepsilon_t$$

it can be written as

$$u_t = k_u + \eta_t$$
$$\sigma_t^2 = k_\sigma + \sum_{i=1}^{\max(P,Q)} \left(a_i + b_i \varepsilon_{t-i}^2\right) \sigma_{t-1}^2$$
$$\varepsilon_t \sim N(0,1)$$

This formula can be coded in vectorial form, allowing a faster computation.

A. *Control variates*

The main idea behind most of the variance reduction techniques, including control variates, is to use the information that you know about the system to get better samples. In this case we will exploit the fact that the value of the variance of the stochastic component of the prices can be theoretically computed. From the model we have

$$y_T^N = \textit{constant} + \sum_{t=1}^{T} h_{T-t+1} u_t$$

If the variance of that expression is computed we obtain

$$\operatorname{var}_t\left(y_T^N\right) = \operatorname{var}(\textit{constant}) + \operatorname{var}_t\left(\sum_{i=t}^{T} h_{T-i+1} u_i\right) =$$
$$= \sum_{t=t}^{T} h_{T-t+1}^2 \operatorname{var}_t\left(u_i\right)$$

where the variance can be obtained iteratively from the GARCH process:

$$\operatorname{var}_t\left(u_i\right) = k_\sigma + \sum_{i=1}^{\max(P,Q)} \left(a_i + b_i\right) \operatorname{var}_t\left(u_{i-1}\right)$$

Initial conditions are the initial volatilities $\sigma_j, j \le 0$. Sample variates are the variances for each scenario of the GARCH model $\left(\sigma_t^r\right)^2$. For each derivative value at date *t*, an improved estimation $\widehat{\theta}_t^*$ of the sample Monte Carlo estimate $\widehat{\theta}_t$ is obtained by applying the formula (see for instance [7]):

$$\widehat{\theta}_t^* = \widehat{\theta}_t + \alpha\left(\operatorname{var}_t\left(u_i\right) - \textit{average}\left(\left(\sigma_t^1\right)^2, \ldots, \left(\sigma_t^R\right)^2\right)\right)$$

where α is the sample correlation coefficient between the derivative values in each scenario θ_t^r and the variances $\left(\sigma_t^r\right)^2$.

B. *Antithetic variates*

The method of antithetic variance attempts to reduce the variance of the estimator by introducing negative dependence between pairs of sample values. Standard antithetic variables are obtaining by negating innovations. In our case, this technique amounts to consider both the scenarios with innovations ε_t and $-\varepsilon_t$. In addition, in order to improve estimates of derivatives at a specific date T_0, the two additional scenarios made by negating ε_t for $t < T_0$ and keeping the original value for $t \ge T_0$, as well as the opposite one, are also considered.

IV. Case Study

In this section we discuss the spot price data collected from European Energy Exchange (EEX) where each day during the year, hourly power contracts are traded on the Day-Ahead Market. We use daily average spot prices from 16/06/2000 to 06/07/2003.

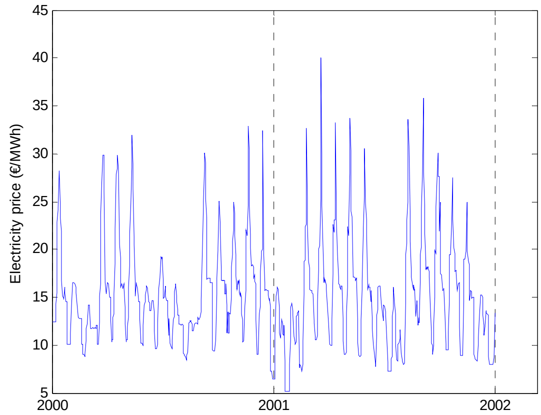

Figure 1.- EEX prices in 2000 and 2001.

It is shown in the Figure 1 the special characteristics of electricity prices, in particular the strong seasonality. The steps of the procedure stated in this paper can be checked with this set of prices (it is assumed that the model described in [5] is fit to this data).

A. *Initial forward curve*

We obtain the forward curve as the average price from a set of price scenarios, generated with the aforementioned model of prices. In addition, the standard deviation of the price scenarios is the option volatility. Both statistics are obtained using variance reduction techniques.

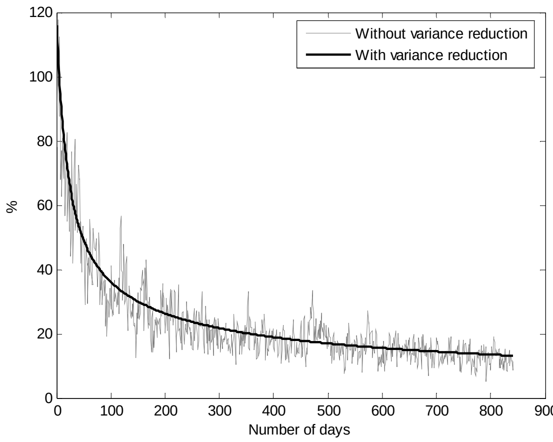

Figure 2.- Estimation with and without variance reduction sampling.

As an instance of its use, Figure 2 compares, with the same number of scenarios, the option volatility curve obtained with the usual estimator and the curve with the variance reduction sampling. It is clear that the estimation of the option volatility is improved when controls and antithetic sampling is used, so it is possible to achieve a good estimation with a reasonable number of scenarios. On the other hand, Figure 3 and 4 shows the forward curve for both monthly and daily product.

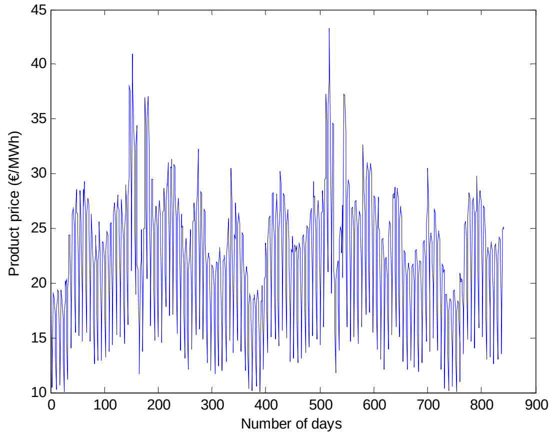

Figure 3.- Forward curve for the daily product.

It can be observed that daily future contracts have a higher volatility than the monthly ones, since the latter are priced with respect to the average price of the whole month.

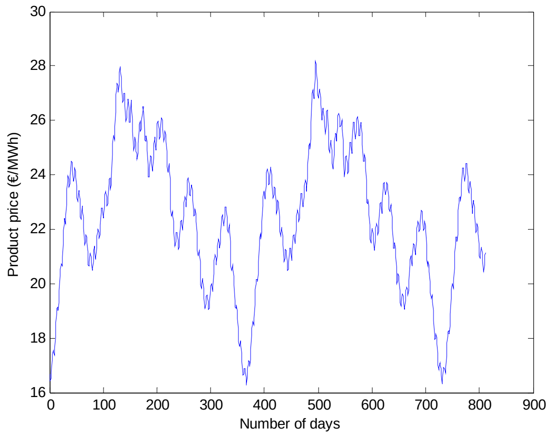

Figure 4.- Forward curve for the monthly product.

B. *Evolved forward curve*

In this step a set of future prices at time *t* are computed. In this case study we have chosen two different sets of valuation dates. On the one hand, we obtain the volatility at the day *t* just before delivery, i.e. *T*-1. On the other hand, we find the volatility at T_0, being this date fifteen days after the beginning of the simulation. The rational for this case is that it is usually required to evaluate the portfolio position in a typically close future date, so we assume that fifteen days after the beginning of the simulation there is a mark-to-market date, and consequently the portfolio risk must be evaluated. Therefore, two sets of future scenarios will be generated: $F^r_{T-1,[T_1,T_2]}$ and $F^r_{T_0,[T_1,T_2]}$. Figure 5 shows the latter set for the first hundred days of simulation for the monthly product. Note that the forward curve changes the first days of simulation, because we are taking into account the first fifteen innovations and their effect tend to disappear in GARCH processes, as the one considered in this paper. This changes are computed using the equation (3).

C. *Computation of the sample statistics*

From the set of forward curve scenarios previously generated, as mentioned in section II, volatility (standard deviation) at *T*-1 and T_0 (in this case study, fifteen days after the beginning of the simulation) are computed. Again, variance reduction techniques are used in order to compute the required standard deviations of the sample. It is represented in the Figure 6 these two volatilities together with the option volatility previously found for the daily future.

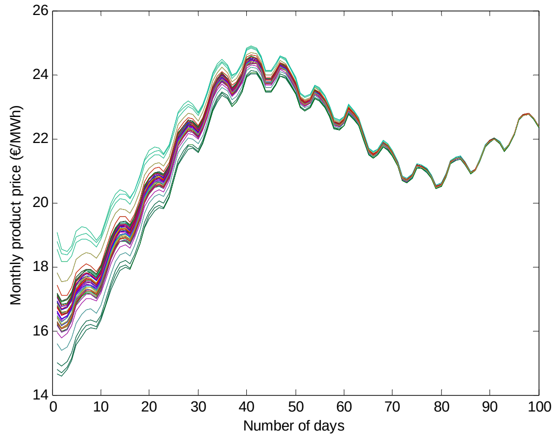

Figure 5.- Forward curve evolution, considering only the 15 first innovations

The curve *maturity* is the future prices at *T*-1, and the *instant* curve is the prices at the day fifteen after the beginning of the simulation. It should be noted that the first day of these two curves is the same, since that point represent the same date of valuation, as well as that option volatility is higher than both volatilities, due to the consideration of a lower amount of innovations in the latter. In addition, it should be noted that the volatilities are mean-reverting (GARCH), as can be observed in their decreasing form. For instance, in a Black-Scholes process, volatilities would be horizontal lines (as the volatility would be constant).

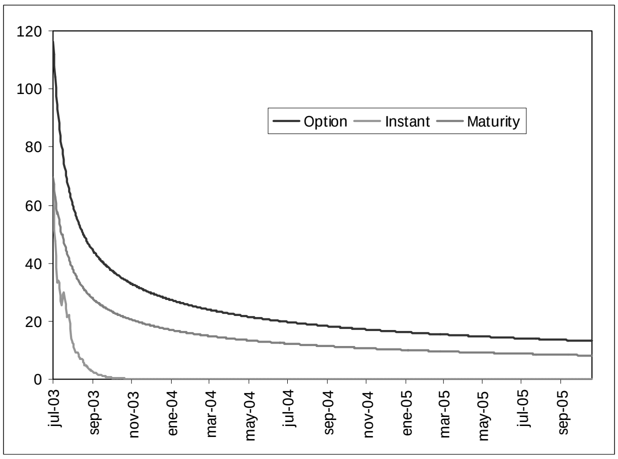

Figure 6.- Volatilities of the monthly product (per cent annualized).

V. CONCLUSIONS

A new methodology for valuating European derivatives of electricity has been proposed. It has been shown that, when non-lognormal distributions are considered (which is frequently the case of electricity prices) analytical expressions for the forward and volatility curves do not exist, or are approximate ones. To cope with the problem, we have proposed a methodology based on Monte Carlo simulation, together with the expression of the forward curve evolution, able to find the forward and volatility curves at any date of the simulation scope. The procedure has been proved with real data of prices of the EEX, showing that it is possible to compute forward and volatility curve even when there are GARCH effects.

VI. APPENDIX A: FORWARD CURVE EVOLUTION DERIVATION

Substituting in the future valuation formula, we obtain

$$F_{0,T} = \mathbf{E}\left[\exp\left(-rt + d_t + y_t^N\right)\right] =$$
$$= \mathbf{E}\left[\exp\left(-rT + d_T + y_0 + \sum_{t=1}^{T} h_{T-t+1}u_t\right)\right] =$$
$$= \exp\left(-rT + d_T + y_0\right)\prod_{t=1}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}u_t\right)\right]$$

Now consider the same future for the next day. Two things have changed: first, the discount factor is $e^{-r(T-1)}$ since there is one day less until the maturity; and the variable u_1 is not stochastic, since it is known. Therefore,

$$F_{1,T} = \exp\left(-r(T-1) + d_T + y_0 + u_1\right)\prod_{t=2}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}u_t\right)\right]$$

Hence

$$\frac{F_{1,T}}{F_{0,T}} = \frac{\exp\left(-r(T-1) + d_T + y_0 + h_T u_1\right)}{\exp\left(-rT + d_T + y_0\right)}\cdot$$
$$\cdot\frac{\prod_{t=2}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}{\prod_{t=1}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]} =$$
$$= \frac{\exp\left(-rT + d_T + y_0\right)\prod_{t=2}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}{\exp\left(-rT + d_T + y_0\right)\prod_{t=2}^{T}\mathbf{E}\left[\exp\left(h_{T-t+1}\left(u_t - \lambda_t\right)\right)\right]}\cdot$$
$$\cdot\frac{\exp\left(r + h_T u_1\right)}{\mathbf{E}\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]} =$$
$$= \exp\left(r + h_T u_1\right)\left\{\mathbf{E}\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]\right\}^{-1}$$

Let us define $u_1^{'} = u_1 - \mu_1$, so that $u_1^{'} \sim N\left(0, \sigma_1\right)$. Besides,

$$\left\{\mathbf{E}\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]\right\}^{-1} =$$
$$= \left\{\mathbf{E}\left[\exp\left(h_T u_1^{'} + h_T\mu_1 - h_T\lambda_1\right)\right]\right\}^{-1} =$$
$$= \exp\left(h_T\lambda_1 - h_T\mu_1\right)\left\{\mathbf{E}\left[\exp\left(h_T u_1^{'}\right)\right]\right\}^{-1} =$$
$$= \exp\left(h_T\lambda_1 - h_T\mu_1\right)\left\{\exp\left(\frac{1}{2}h_T^2\sigma_1^2\right)\right\}^{-1}$$

Putting these equations all together we obtain

$$\frac{F_{1,T}}{F_{0,T}} = \exp\left(r + h_T u_1\right)\left\{\mathbf{E}\left[\exp\left(h_T\left(u_1 - \lambda_1\right)\right)\right]\right\}^{-1} =$$
$$= \exp\left(r + h_T u_1\right)\exp\left(h_T\lambda_1 - h_T\mu_1\right)\left\{\exp\left(\frac{1}{2}h_T^2\sigma_1^2\right)\right\}^{-1} =$$
$$= \exp\left(r + h_T\lambda_1 - \frac{1}{2}h_T^2\sigma_1^2\right)\exp\left(h_T\left(u_1 - \mu_1\right)\right)$$
$$= \exp\left(r + h_T\lambda_1 - \frac{1}{2}h_T^2\sigma_1^2\right)\exp\left(h_T u_1^{'}\right)$$

Then, it is clear that

$$\ln F_{t+1,T} - \ln F_{t,T} = r + h_{T-t+1}\lambda_t - \frac{1}{2}h_{T-t+1}^2\sigma_t^2 + h_{T-t+1}u_t^{'}$$

To better understand the problem, this equation can be thought of as the discrete form of

$$d\ln F_{t.T} = \left(r + h_{T-t}\lambda_t - \frac{1}{2}h_{T-t}^2\sigma_t^2\right)dt + h_{T-t}\sigma_t dz_t$$

From the Ito's lemma we have

$$\frac{dF_{t,T}}{F_{t,T}} = \left(r + h_{T-t}\lambda_t\right)dt + h_{T-t}\sigma_t dz_t$$

which is a very similar expression to the Black-Scholes one.

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