

Discrete decisions, plants revenues and prices in electricity markets

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Abstract

In power systems models, there are some variables of discrete nature, such as those modeling the state of the power units, started up or stopped. As a result, the marginal cost of operation is, in general, insufficient to pay the costs associated with these variables (eg, start up cost). Therefore, even if highly idealized conditions, as perfect competition, are assumed, the marginal cost is not expected to be the electricity price.

From the point of view of mathematics, the problem arises due to the non convex nature of the function cost. Currently, there is no general method in the literature to estimate the prices in market equilibrium, not even being possible to prove its existence. This paper propose some alternatives to calculate the value of these prices, when the effect of the discrete variables in the power market model is considered.

Keywords: mathematical programming, optimization methods, power system economics, power system modelling

1. Nomenclature

Parameters

- λ_{psl}^* Resultant marginal price in period p , subperiod s , and load level l of the market model MM with continuous variables.
- MC_{psl}^{e*} Resultant marginal cost for firm e in period p , subperiod s , and load level l of the market model MM with continuous variables.
- P_{psl}^{e*} Resultant production for firm e in period p , subperiod s , and load level l of the market model MM with continuous variables.
- P_{psl}^{u*} Resultant production for power unit u in period p , subperiod s , and load level l of the market model MM with continuous variables.
- $P_{psl}^{e, fwd}$ Forward contracts for firm e in period p , subperiod s and load level l .
- $C(\cdot)$ Cost function.
- $C^*(\cdot)$ Exterior linear approximation of the cost function.
- K_{psl}^e Penalty factor in the Auxiliary Problem for production deviation of firm e in period p , subperiod s and load level l .
- θ_{psl}^e Conjectural Price Response for firm e in period p , subperiod s , and load level l ,
- $$\theta_{psl}^e = -\frac{\partial \lambda_{psl}}{\partial P_{psl}^e}.$$
- D_{psl} Demand paying the market price in period p , subperiod s , and load level l .

Variables

- λ_{psl}^{total} Total marginal price paid by the demand, after considering the discrete nature of the ‘start-up’ and ‘shut-down’ variables, in period p , subperiod s , and load level l .
- AR_{psl}^u Additional revenue for power unit u in period p , subperiod s , and load level l .
- AR_{psl}^e Additional revenue for firm e in period p , subperiod s , and load level l .

MC_{psl}^e	Marginal cost for firm e in period p , subperiod s and load level l .
P_{psl}^e	Production of firm e in period p , subperiod s , and load level l .
P_{psl}^u	Production of power unit u in period p , subperiod s , and load level l .
$\Delta_{psl}^{1,u}$	Production deviation up of power unit u in period p , subperiod s and load level l , between the production level resulted from the market model MM with continuous variables and the plausible production using binary variables.
$\Delta_{psl}^{2,u}$	Production deviation down of each power unit u in period p , subperiod s and load level l , between the production level resulted from the market model MM with continuous variables and the plausible production using binary variables.
$\Delta_{psl}^{1,e}$	Production deviation up of each firm e in period p , subperiod s and load level l , between the production level resulted from the market model MM with continuous variables and the plausible production using binary variables.
$\Delta_{psl}^{2,e}$	Production deviation down of each firm e in period p , subperiod s and load level l , between the production level resulted from the market model MM with continuous variables and the plausible production using binary variables.

2. Introduction

When market models are used to represent power systems, it must be borne in mind how the power units are modelled in order to carry out a precise analysis of the results. These models are usually based on optimization problems, aimed at finding the optimal generation scheduling when maximizing the social surplus. Nevertheless, they show some difficulties calculating the market prices. When the problem is continuous, it is possible to determine the marginal prices from the dual variables in the optimization problem. However, when there are some discrete variables in the problem, the results from the marginal theory [1] are not applicable in a straight way, and it is necessary to use some kind of complementary method as in [2] or [3]. In [2], each discrete decision is priced as a separate commodity. The authors solve a MIP to find the optimal allocation. Next, they remove the integrality constraints and insert equality constraints that force the integer variables to assume their optimal values in the resulting LP. The dual variables of the equality constraints added are the prices for the discrete decisions. Pricing scheme in [3] also considers payments for the lost profit of some power units willing to produce more than the production level they have been assigned by the dispatch algorithm.

The sequel of this paper is organized as follows. The next section explains why the price signal derived from common electricity models is not enough to pay all the production costs to the power units. In section 3, we propose a method to adjust the price, resulting from a continuous representation of the on/off variables in the model, with a plausible dispatching of the power units, resulting from a discrete representation of the on/off variables. Section 4 is devoted to present a real size case study, inspired in the Spanish power system. In the last section, we finish with some conclusions.

2. Price signals

The theoretical background of this paper has as starting point the work developed in [4]. In Fig.1, the solid curve shows an idealized short-term cost function. It represents the total dispatch costs as a function of the demand, for some hour. If the market is cleared by means of a power system model minimizing total system costs, the marginal cost is the slope of the cost function in the point where it intersects the demand curve. This value is not enough to pay some of the infra-marginal units' production costs. For example, it does not take into account the start up costs.

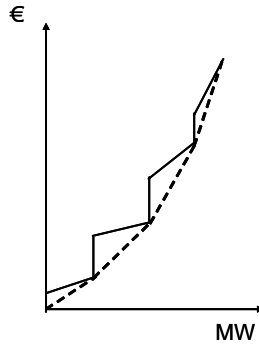


Fig.1 – Cost Function and Envelope.

The dashed curve in Fig.1 represents the envelope of the cost function. It is a piecewise linear convex curve and it can be defined as an exterior linear approximation of the cost function. An approximation to the envelope is to represent the on/off variables of the power units as continuous variables. If the market is cleared with this new cost function, the resulting marginal cost in this case is the slope of the piecewise linear cost function, the dashed curve in Fig.1. This price value represents the average cost of the marginal power unit. It is enough to pay all the infra-marginal units, but it is not sufficient for the marginal unit to recover all its ‘start up’ costs. Though the on/off variables are now continuous in the model, the marginal unit can not start up partially in real operation. Moreover, there could be some power units with lost profits, willing to produce more than the production level they have been assign by the dispatch algorithm. An alternative to solve this problem is to make additional payments, aside the market price [4].

Somehow, this is the procedure in practice, through subsequent markets after the day-ahead market (i.e. intraday, balancing, reserves...). These markets allow agents to adjust their productions to the market deviations in the short-term and, at the same time, marginal units to make up the production costs. Thus, the solution of this problem should give information about the additional costs and incomes in regard to these markets, closer to real time.

3. Proposal for simulation

The simulation proposed here has as starting point the model presented in [5] as a quadratic optimization problem, to which we will refer as Market Model (MM). This model is briefly presented here to make the reading easier:

Market Model (MM)

$$\min \sum_{e,p,s,l} \left[C(P_{psl}^e) + \frac{1}{2} \theta_{psl}^e \cdot (P_{psl}^e - P_{psl}^{e, fwd})^2 \right]$$

subject to:

$$\sum_e P_{psl}^e = D_{psl} \quad \forall p, s, l$$

and other technical and economical constraints

When the cost function is convex, it can be proved that the optimal conditions of MM are the market clearing equilibrium equations [5]. That is, marginal cost and marginal revenue are equalized for every company, and total generation equals demand. Moreover, demand balance equality multiplier is the clearing price. The variables θ_{psl}^e represent the market power potential of every agent. In perfectly competitive situations they are zero. In oligopolistic situations they are positive magnitudes.

In addition to the conceptual problems explained in the section above there is also a methodological difficulty relating to the fact that MM is a quadratic problem, and if

discrete variables are considered it is almost on the feasible limit with the current optimization programs. This is even truer if stochastic programming is addressed as in [6], although the present paper does not tackle the stochastic case. Thus, it is interesting to explore alternative simulation methods to give a good approximation to the electricity price when we consider discrete variables.

The proposal for simulation starts solving MM, modelling the ‘start up’ and ‘shut down’ decisions for the power units as continuous variables. To simulate the market equilibrium the discrete nature of the on/off variables is considered (that is, after intraday, reserve markets and the like are cleared), we set out two main alternatives:

1. Solve the whole model MM with discrete variables in a straight way.
2. Solve an auxiliary problem

Auxiliary Problem (AP)

$$\min \sum_{e,p,s,l} \left[C(P_{psl}^e) + \theta_{psl}^e \cdot (P_{psl}^{e*} - P_{psl}^{e, fwd}) \cdot P_{psl}^e + K_{psl}^{e*} \cdot (\Delta_{psl}^{1,e} + \Delta_{psl}^{2,e}) \right]$$

subject to:

$$P_{psl}^e = P_{psl}^{e*} + \Delta_{psl}^{1,e} - \Delta_{psl}^{2,e} \quad \forall e, p, s, l$$

$$\sum_e [\Delta_{psl}^{1,e} - \Delta_{psl}^{2,e}] = 0 \quad \forall p, s, l$$

$$\Delta_{psl}^{1,e}, \Delta_{psl}^{2,e} \geq 0$$

and other technical and economical constraints

Perfect and oligopolistic competition can be simulated by plugging appropriate values of the conjectural price response θ_{psl}^e . The auxiliary problem has the advantage of being linear. The aim of this auxiliary optimization problem is to obtain a feasible production for the power units adjusted to the market price and the production levels obtained as a result of the model with continuous variables, which represents better the signal for the power units to recover their production costs.

The rationale of the above model is to mimic the temporal development of the electricity market. Bids in the daily market are portfolio bids that can be thought as derived from the relaxed cost function, as the exact one when integrality constraints are considered is not a very different one. This fact is even more relevant when uncertainty in the market is considered, which in any case should lead to the companies to a “fuzzier” view of their competitors bids than otherwise.

As result of the daily market clearing, a position (approximated by the relaxed model output) is assumed by each company. Then, the auxiliary problem can be thought as a model for the intraday or balancing markets. Penalty parameters K_{psl}^e can be thought to measure the liquidity of the market or the agents’ propensity to trade.

In both cases, either by solving the whole model MM with discrete variables in a straight way, or solving the auxiliary problem, the estimation of the total energy price to be paid by the demand is calculated in the same way. The cost difference between the continuous and the discrete solutions is calculated. That cost difference is awarded to the power units as an additional payment.

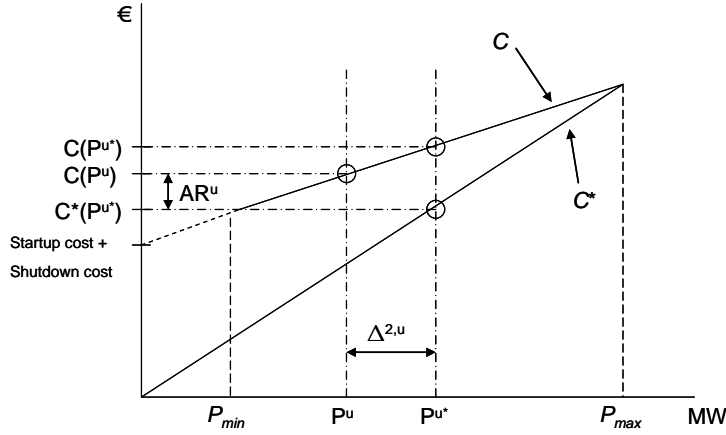


Fig.2 – Additional Revenues with perfect competition.

In order to gain some insight, let us consider Fig.2. The real cost function C of a marginal power unit u and its exterior linear approximation C^* are shown. Modelling the ‘start up’ and ‘shut down’ decisions for the power units as continuous variables, the resultant production for the power unit u after solving MM is P^{u*} and its production cost is $C^*(P^{u*})$. The marginal price λ^* is enough to pay all the production costs $C^*(P^{u*})$. But, as stated before, this is not the real production cost. The real cost of producing P^{u*} is $C(P^{u*})$. Even more, the production level P^{u*} could not be feasible.

In Fig.2 is shown that, after solving the auxiliary problem AP or after solving the whole model MM with discrete variables in a straight way, the final production of power unit u is P^u , and its real production cost is $C(P^u)$. Then, under the assumption of a perfectly competitive market, unit u should receive an Additional Revenue AR^u for the additional production cost. With imperfect competition the situation can be seen as analogous to the perfect competition case, but considering that each firm acts with an *increased cost* $C(P_{psl}^e) + \frac{1}{2}\theta_{psl}^e (P_{psl}^e - P_{psl}^{e, fwd})^2$. This fact can be seen in the objective function of MM. The difference is that in this case the quadratic term can not be separated for each power unit, but it is a portfolio effect. However, it is possible to linearize this term in order to define the following quantities:

$$AR_{psl}^e = C(P_{psl}^e) - C^*(P_{psl}^{e*}) + \theta_{psl}^e \cdot (P_{psl}^{e*} - P_{psl}^{e, fwd}) \cdot (P_{psl}^e - P_{psl}^{e*})$$

$$AR_{psl}^u = C(P_{psl}^u) - C^*(P_{psl}^{u*}) + \theta_{psl}^e \cdot (P_{psl}^{e*} - P_{psl}^{e, fwd}) \cdot (P_{psl}^u - P_{psl}^{u*})$$

$$\lambda_{psl}^{total} = \lambda_{psl}^* + \frac{\sum_e [C(P_{psl}^e) - C^*(P_{psl}^{e*}) + \theta_{psl}^e \cdot (P_{psl}^{e*} - P_{psl}^{e, fwd}) \cdot (P_{psl}^e - P_{psl}^{e*})]}{D_{psl}}$$

Then, λ_{psl}^{total} is the estimated price paid by the demand, in order to cover all the costs. Power units should be paid the ‘‘relaxed’’ (day-ahead) price λ_{psl}^* times the ‘‘relaxed’’ production P_{psl}^{u*} plus an additional ‘‘intra-daily’’ revenue AR_{psl}^u . It must be pointed out that, even if the firm e does not deviate from P_{psl}^{e*} , i.e. $\Delta_{psl}^{1,e} - \Delta_{psl}^{2,e} = 0$, still it can receive AR_{psl}^e , because of possible differences between the ‘‘integer’’ and ‘‘relaxed’’ costs.

4. Case study

A real size case has been considered. Although the case is based on the Spanish system, used data and assumptions are not necessarily real ones, so results presented should not be interpreted in any way as representative of a real situation. They provide a test-bed to illustrate the application of the above presented simulation.

The time horizon of the study is one year. One period per month has been considered. Each period is divided into two subperiods to contemplate working and non-working days, and each subperiod is divided into three different types of load levels (peak, shoulder & off-peak). So, the study considers 72 load levels.

Only inelastic demand has been considered. The maximum peak demand is close to 36000 MW and the yearly energy demand is 225000 GWh. The generation structure of the system is shown in Table 1.

Area	Generating Company	Nuclear	Coal	Gas	Hydro	Pumping
a	1	3498 (4)	5600 (16)	3405 (12)	3563 (12)	1431 (6)
	2	3200 (4)	1212 (5)	5110 (12)	5282 (6)	1664 (4)
	3	710 (1)	1946 (7)	1336 (5)	1332 (4)	216 (1)
	4	154	1500 (5)	370 (1)	314 (1)	107 (1)
	5	-	875 (5)	731 (2)	497 (2)	360 (1)
	6	-	-	1126 (3)	-	-
	7	-	90	973 (2)	-	-

Table 1. System Dimensions. Installed Power (MW) and Number (in parentheses) of Groups Owned by each generating company

Technical constraints must also be modelled. In particular, constraints related to the start up, coupling and stoppage of thermal units, minimum productions of thermal units, and water management, among others, have been included. Water management is allowed within periods but not among different periods.

In Table 2 we show the standard deviation, maximum and minimum values for the production deviation $\Delta_{psb}^{1,e} - \Delta_{psb}^{2,e}$. The real production costs and the Quadratic costs are shown as a percentage of the result in the MM with continuous variables. The additional payments that the demand should afford are presented as a percentage of the electricity price resulted in the MM with continuous variables.

		Perfect Competition		Imperfect Competition			
		Discrete MM	AP	Discrete MM	AP		
Deviation penalty		-	$K_{psb}^{e,*} = CM_{psb}^{e,*}$	-	$K_{psb}^{e,*} = CM_{psb}^{e,*}$	$K_{psb}^{e,*} = 2 \cdot CM_{psb}^{e,*}$	$K_{psb}^{e,*} = 10 \cdot CM_{psb}^{e,*}$
Production deviation $\Delta_{psb}^{1,e} - \Delta_{psb}^{2,e}$ (MWh)	std	3956	640	9780	4334	4344	2051
	Max	19120	6256	67936	52416	52416	19278
	Min	-16700	-6256	-85888	-34524	-31560	-21840
Production cost $C(P_{psb}^e)$ (%)	Total	0.74	0.91	0.63	0.86	0.86	0.91
	$\frac{\lambda_{psb}^{total} - \lambda_{psb}^*}{\lambda_{psb}^*}$ (%)	Total	0.205	0.251	0.176	0.234	0.235

Table 2. Results (% related to the continuous case)

With the Auxiliary Problem, the production level of each company is closer to the solution of the MM with continuous variables, than solving the whole model MM with discrete variables in a straight way. As it is expected, this smaller deviation produces a higher production cost, because the problem has an additional constraint.

In the Auxiliary Problem, $K_{psb}^{e,*}$ is a key parameter. With perfect competition, for example, if $K_{psb}^{e,*} = 0$ the solution for the AP model and the discrete MM would be the same. That is to say, if deviating from the production assigned in the continuous MM is not penalized, the solution of the subsequent markets (AP model) would lead to the solution of the discrete MM. Then, $K_{psb}^{e,*}$ can be interpreted as a measure of the liquidity of the subsequent markets. The greater the value of $K_{psb}^{e,*}$, the smaller the liquidity.

With imperfect competition, three different values of the deviation penalty have been considered. Due to this reduced liquidity, the total production cost is greater with the AP, and greater, the greater is the value of $K_{psb}^{e,*}$. If the value of $K_{psb}^{e,*}$ is doubled, from $CM_{psb}^{e,*}$ to $2 \cdot CM_{psb}^{e,*}$, the solution of the APIC model does not change significantly. If the value increases to $10 \cdot CM_{psb}^{e,*}$, the total production cost increases in 0.05% while the production standard deviation reduces to the 50%, compared with $K_{psb}^{e,*} = CM_{psb}^{e,*}$. Thus, it can be considered that the lack of liquidity in the subsequent markets does not produce a dramatic increase in the production costs.

In the discrete MM, the firms do not have the imposition to follow their production resulted with the continuous MM. With the AP model, firms in order to minimize their costs try to achieve the discrete MM solution, but they are penalized for their deviations from their production resulted with the continuous MM. This can be interpreted as if they would like to change in the subsequent markets, their positions achieved in the day-ahead market. They can change their positions, but not as much as they would like as long as they are penalized for their deviations (they have to buy energy in the subsequent markets to change their positions).

5. Conclusions

This paper proposes an electricity market simulation procedure that allows to estimate total electricity price, plants productions that fulfil integrality constraints and revenues from day-ahead and intraday markets for each generating unit. The authors are working to extend this procedure to the stochastic case.

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