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Computation and decomposition of marginal costs for a GENCO in a constrained competitive Cournot equilibrium

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Abstract: This paper presents a methodology to compute and decompose marginal costs for a Generation Company (GENCO) in a competitive framework. The methodology is based on a Cournot market equilibrium model. The aim is to provide a useful tool for GENCOs so that they can manage their resources in an optimal way, helping them on their decision making processes, asset valuations or contract assessments.

Keywords: Cournot equilibrium, energy markets, marginal cost.

I. INTRODUCTION

During the last years, deep reorganisations have been taking place in the electricity sectors in many countries. There is a pronounced trend towards liberalisation and decentralisation. In this context, new tools are required for companies to assume their new role. Mathematical models to simulate energy markets are a fundamental part. In a number of them, the model proposed for Antoine Agustin Cournot in 1838 [1] has been applied. A complete characterisation of the Cournot equilibrium can be found in [2]. One of the first works using Cournot model to represent electricity markets can be found in [3]. In [4], [5] and [6] Cournot equilibrium is applied to hydrothermal co-ordination. In [7] and [8] the same methodology is used taking into account transmission network.

In a competitive framework, marginal costs become a very important tool for decision-making in a GENCO. To be more specific, the companies will try to have the same marginal cost along the time.

It is also of great interest to decompose marginal costs into their different components. By doing so, companies are prepared to use the terms they consider appropriate for different situations (contract-valuing, water value, bid-construction...) and different time scopes. For example, different marginal costs must be taken into account when assessing different contracts: long-term or short-term contracts, peak-hour contracts or valley-hours contracts, contracts for a little or large amount of energy. In [9] a decomposition of marginal costs in a centralised framework with transmission network is performed.

This paper proposes a methodology to identify marginal costs for GENCOs. This will be an important help for them to manage their resources in an optimal way. To represent market equilibrium, a Cournot model is used.

II. DESCRIPTION OF THE EQUILIBRIUM MODEL

The aim of this section is to describe the model proposed to represent the market equilibrium. First, theoretical results are developed in the case where only one load level is considered and no technical constraints are taken into account. In section III, the whole system with constraints and different load levels is described.

A. Marginal costs in a centralised framework.

In a centralised framework, the dispatch is performed through the minimisation of the total cost of the system (see [10]):

$$\begin{aligned} & \underset{P_g}{mín} \quad \sum_{g=1}^{G} C_g\left(P_g\right) = C\left(P_1, P_2, ..., P_G\right) \\ & \text{subject to} \qquad D = \sum_{g=1}^{G} P_g \qquad : \quad l \\ & \text{and} \qquad \text{Technical constraints} \end{aligned} \tag{1}$$

Where P_g is the production of each generation group ($g=1,2,...G$). $C_g(P_g)$ is the cost of every group as a function of its generation level. $C(P_1, P_2,..., P_G)$ is the total cost function and D is the demand, that behaves as an inelastic competitor.

In a centralised dispatch, there is only one marginal cost for the whole system. This marginal cost is the cost of generating an extra MWh. By definition, it corresponds to the dual variable l of the demand constraint:

$$l = \frac{\partial C\left(P_1, P_2, ..., P_G\right)}{\partial D} \tag{2}$$

B. Cournot market equilibrium.

The market equilibrium model proposed by Cournot has been widely used to represent the strategic behaviour of a number of companies competing in a market. In this model, every firm ($e=1,2,...E$) participates in the market by generating a certain quantity P_e that each company chooses from the maximisation of their profits. Each firm is the owner of a number of generation groups, so $P_e = \sum_{g \in e} P_g$.

Besides, the demand is modelled as a decreasing function of the price l. In many studies, the function is a linear one, being a_0 its slope.

$$D = D^0 - l \cdot a_0 \tag{3}$$

This way, the existence and uniqueness of the solution is guaranteed if the cost function is continuous and convex.

The variables for the generation and the demand are linked through the power balance equation:

$$D = \sum_{e=1}^{E} P_e \tag{4}$$

The benefit B_e that a firm obtains when it is being remunerated at the marginal price is:

$$B_e = I_e(l, P_e) - C_e(P_e) = l \cdot P_e - C_e(P_e) \tag{5}$$

Being $I_e(l, P_e)$ the income function for the firm.

The Cournot equilibrium is obtained through the maximisation of the profit for all the agents, with respect to their quantity. Thus:

$$\frac{\partial B_e}{\partial P_e} = 0 = l + P_e \cdot \frac{\partial l}{\partial P_e} - \frac{\partial C_e(P_e)}{\partial P_e} \tag{6}$$

It can be proved that this problem is equivalent to the following optimisation problem:

$$\begin{aligned} &\min_{P_e, D} \quad \sum_{e=1}^{E} \bar{C}_e(P_e) - U(D) \\ &\text{s.t.} \quad D = \sum_{e=1}^{E} P_e \qquad : \quad l \\ &\qquad \text{Technical Constraints} \end{aligned} \tag{7}$$

Where $\bar{C}_e(P_e)$ denotes a term called *effective cost function* and $U(D)$ is the utility function for the demand.

$$\bar{C}_e(P_e) = C_e(P_e) + \frac{P_e^2}{2 \cdot a_0} \tag{8}$$

$$U(D) = \int_0^D l(D)\, dD = \frac{1}{a_0} \cdot \left(D \cdot D_0 - \frac{D^2}{2} \right) \tag{9}$$

Under the hypothesis of continuity and convexity in the cost functions, it can be proved that this optimisation problem is equivalent to the Cournot equilibrium problem. The first and second order conditions are the same for both approaches. Note that the dual variable of the demand constraint is the marginal price of the system.

C. Marginal income and marginal cost.

As it will be shown, it is not clear how to define the marginal cost for a GENCO competing in a market. It depends on the use given to this cost. First, the theoretical marginal cost (the derivative of the cost function) will be calculated. Then, it will be decomposed into different terms and the impact of every one in the operation of a GENCO will be discussed.

At the equilibrium, the marginal income and the theoretical marginal cost become equal for every firm.

$$\frac{\partial C_e(P_e)}{\partial P_e} = l - \frac{P_e}{a_0} = \frac{\partial I_e(l, P_e)}{\partial P_e} \tag{10}$$

There are two important differences between this result and that obtained from a centralised framework. On the one hand, these theoretical marginal costs are not equal to the dual variable of the demand constraint. On the other hand, there is one marginal cost for every company, whereas in the centralised case there is a unique value for the whole system.

It is very important for a GENCO to know its marginal costs, and quantify how different situations are modifying them. This knowledge allows the company to improve its operation. If every component of the theoretical marginal cost is known, the GENCO will be able to define its actual marginal cost for different situations.

III. MARGINAL COSTS DECOMPOSITION

A time horizon with B load levels will be supposed. Within each level, all the operation parameters will be considered as constants. Duration for each load level will be considered to be one hour.

The optimisation problem to be solved in order to calculate the market equilibrium, as follows:

$$\min_{P_{be}, D_b} \quad \sum_{b=1}^{B} \sum_{e=1}^{E} \bar{C}_e(P_{be}) - U(D_b) \tag{11}$$

Subject to:

$$D_b = \sum_{e=1}^{E} P_{be} \qquad : \quad l_b \tag{12}$$

$$P_{bg} \le \bar{P}_g \qquad : \quad p_{bg}^{M} \tag{13}$$

$$\sum_{b=1}^{B} P_{bg} \ge S_g \qquad : \quad p_g^{C} \tag{14}$$

$$\sum_{b=1}^{B} P_{be} \ge Q_e \cdot \sum_{b=1}^{B} D_b \qquad : \quad p_e^{Q} \tag{15}$$

$$P_{be} \ge QL_e \cdot D_b \qquad : \quad p_{be}^{L} \tag{16}$$

Constraint (12) represents the balance between generation and demand and equation (13) represents the maximum power $\bar{P}_g$ for generation groups. Equation (14) implies a minimum production S_g for a certain number of groups during the time horizon. Constraint (15) responds to the need of some companies to reach a minimum global market share Q_e. Finally, equation (16) represents a minimum market share QL_e that must be reached for every load level.

Dual variables are indicated at the right side of the equations. All of them will be positives, with the exception of the one of the constraint (13). This is because it is a minimisation problem.

A. Analysis with only operational constraints.

First, the case with only operational constraints (energy balance and maximum power limits) will be studied. The simplest situation arises when the company has a continuous cost function with a continuous derivative (Figure III.1). In this case, the marginal income (or marginal cost) is the derivative of the cost function.

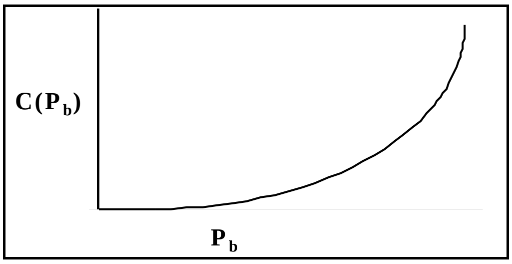

Fig. III.1 Cost function with continuous derivative

Marginal cost function is then continuous and matches up with the equation (10). There is no possible distinction between theoretical and actual marginal cost in this situation.

But the cost function of a company is often approximated to a convex linear function with non-continuous derivative. In particular, this usually happens in large size systems. As shown in Figure III.2, each segment corresponds to a generation group with a variable cost (supposed to be constant) which is equal to the derivative in that segment.

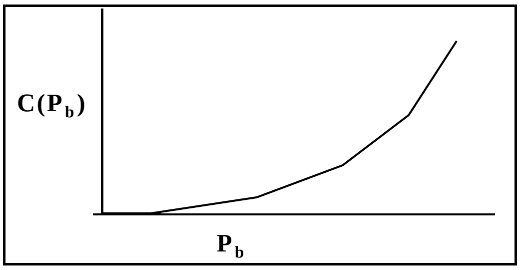

Fig. III.2 Cost function with discontinuous derivative

With this representation, the actual marginal cost function for a company is a piece-wise function. Equation (10) gives the marginal income (or theoretical marginal cost) and that is, in general, different from the marginal cost. So it is necessary to identify the marginal cost from the marginal income. Figure III.3 represents the marginal cost *M* as a function of the company's production level.

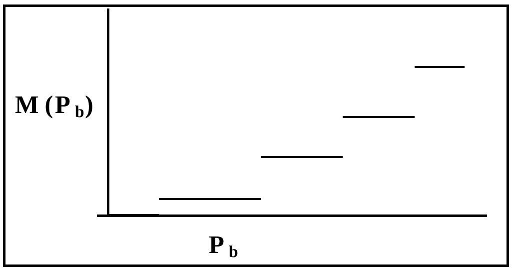

Fig. III.3 Marginal cost function

In general, three situations are possible for a company:

- A group generates below its maximum power. In this situation, the marginal income and the marginal cost are equal and correspond to the variable cost of the group.
- Some groups generate at their maximum power while others do not produce. In this case, the marginal income is between the variable cost of the most expensive group that produces and the variable cost of the cheapest group that does not produce.

  It is not easy to define the marginal cost in these situations, because it depends on the use that the company is going to give it. For example, the cost of generating extra energy will be the variable cost of the cheapest group that does not produce. Nevertheless, the cost of the last energy generated is the variable cost of the most expensive group that produces.
- All the groups generate at their maximum capacity. The marginal income is higher than all the variable costs of the company's generation groups. In this case, if any marginal cost can be defined, it will correspond to the most expensive variable cost.

In any of the three situations, if $c_{\hat{g}}$ is the variable cost of any group $\hat{g}$ that produces in level *b*, the marginal income *J* given by equation (10) for the firm which owns of the group, will verify:

$$J_{be} = c_{\hat{g}} - p^{M}_{b\hat{g}} \qquad (17)$$

Note that $p^{M}_{b\hat{g}}$ is non-positive, so the marginal income will be higher than (or equal to) the variable cost of the groups that generate.

B. Analysis with all the constraints.

The case with all the constraints will now be developed. It will be shown how this modifies the relationship between the marginal income and the costs of the groups. For simplicity, the maximum power constraint will be supposed not to be active.

B.1 Groups with an incentive to produce.

There are situations (often regulatory ones, e.g. the protection to renewable generation, or autochthonous fuels) in which some groups (with variable cost c_i) receive an incentive g_i to produce.

In this case, when these groups are generating at their maximum power, or are not generating, the marginal cost does not vary.

On the contrary, when one of these groups is generating below its maximum power, the marginal income obtained is:

$$J_{be} = c_i - g_i \qquad (18)$$

Thus, the group behaves as if it had a real cost of c_i - g_i. For the company, this means that it has to operate the group as if it had this actual cost.

In this case, the marginal cost must consider this incentive. Thus, the marginal cost is equal to the marginal income.

B.2 Minimum production forced in some groups.

Sometimes, some groups are forced or interested (due to technical, political or strategic reasons) in generating above a certain production level. Typical reasons are incidents (congestion or voltage problems) in the transmission network.

Again, when all these groups are generating at their maximum power, or are not generating, the marginal cost remains without any variation.

But in the load levels in which one of these groups (that will be denoted by $g*$) is generating below its maximum power, the marginal income is:

$$J_{be} = c_{g*} - p^{C}_{g*} \qquad (19)$$

Groups behave as if they had a real cost decreased by the dual variable of its minimum production constraint. As in *B.1*, these groups should be operated as if their actual cost were decreased by this dual variable.

B.3 Minimum global market share.

Other common situation in a competitive framework appears when a company tries to assure a minimum market share during the considered time horizon. Typically, this behaviour can arise from a long term strategy.

Then, if $\tilde{g}_{be}$ is the most expensive group (with a production cost $\tilde{c}_{be}$) of the firm e that generates at level b, the marginal income perceived by the company is:

$$J_{be} = \tilde{c}_{be} - p^{Q}_{e} \qquad (20)$$

The effect is that all the groups owned by the company behave as if they had a real cost decreased by the dual variable of the minimum share constraint. This behaviour takes place during the whole time horizon.

A generation company must take into account this effect when operating their groups. For instance, when building its bid function. If this dual variable is not taken into account, the company will not reach the target market share.

Other situation arises when the company have to assess a contract. The behaviour in this case will depend on the characteristics of the contract. This is: if the energy sold (or bought) counts towards the considered share; if it is a bilateral contract, etc.

Consequently, it does not make sense to define the marginal cost in these situations. It has to be considered for every specific use.

B.4 Minimum market share for each load level.

Again because of long term policies, a company could also be interested in a market share always higher than a minimum value in every load level. The result obtained would be:

$$J_{be} = \tilde{c}_{be} - p^{L}_{be} \qquad (21)$$

In the load levels in which this constraint is activated (the dual variable is positive) the company should operate all their groups with an actual cost decreased by the value of this dual variable. Otherwise, this target share would not be reached in those load levels.

The same considerations as in B.3 should be taken into account about the marginal cost and its use.

C. General expression

Based on the marginal income given by equation (10), a general expression can be obtained to identify the different terms that can be included in the marginal cost. The marginal cost can be expressed as follows:

$$J_{be} = \tilde{A}_{be} - Z_{be} \qquad (22)$$

In this expression, $\tilde{A}_{be}$ denotes the *apparent cost* of group $\tilde{g}_{be}$. Z_{be} includes all the dual information that affects to the marginal costs.

If a group g has an apparent cost A_g in a load level, it means that its behaviour in this load level would be the same as the behaviour of a group with that variable cost. Three situations can arise:

- If the group $\tilde{g}_{be}$ has an incentive to produce, $\tilde{A}_{be} = \tilde{c}_{be} - g_{\tilde{g}}$.
- For a group $\tilde{g}_{be}$ with a minimum production constraint $\tilde{A}_{be} = \tilde{c}_{be} - p^{C}_{\tilde{g}}$.
- If the group has neither incentive nor constraint, $\tilde{A}_{be} = \tilde{c}_{be}$

On the other hand, Z_{be} includes all the dual information that affects to the marginal costs:

$$Z_{be} = p^{Q}_{e} + p^{L}_{be} + p^{M}_{b\tilde{g}} \qquad (23)$$

Note again that all the dual variables are non-negatives except $p^{M}_{b\tilde{g}}$.

IV. CASE STUDY

The case study represents a system with two generation companies. GENCO x owns three generation groups: *1*, *2*, and *3*. GENCO y owns groups *4* and *5*. Two load (p and v) levels will be considered.

Table IV.1 shows the installed generation capacity and the cost for every generation group.

Table IV.2 shows the demand parameters and the duration considered for the load levels. Demand is considered to be elastic, as shown in equation (3).

Four cases have been represented. The first one (case *a*) corresponds to a market equilibrium with only operational constraints.

In the second one (case *b*), market share constraints have been implemented: a global market share of 70%, and a market share of 66% in every load level. Both constraints are for company x.

The third case (*c*) includes a constraint of a minimum generation of 5 GWh for group *3* (owned by company *x*).

Finally, the last case (d*)* includes both constraints of market share and minimum generation, as described in the second and third case.

	1	2	3	4	5
Generation capacity (MW)	1000	500	300	800	400
Production cost (€/MWh)	10	20	30	15	25

Table IV.1 Characteristics of generation groups

	p	v
Duration (h)	1	1
Slope a_0 (GW/(€/MWh))	10	15
Demand for price 0 (MW)	5000	4000

Table IV.2 Duration and demand

Productions (*P*) for every group, demands (*D*), prices (*l*) obtained are shown in table IV.3 for both load levels (*p* and *v*).

Table IV.4 shows the marginal income obtained for both generation companies in both load levels. This marginal income (or theoretical marginal cost) has been computed as described in equation (10).

		a	b	c	d
x	P_{1p} (MW)	1000	1000	1000	1000
	P_{1v} (MW)	1000	1000	1000	1000
	P_{2p} (MW)	99	500	40	500
	P_{2v} (MW)	0	87	0	84
	P_{3p} (MW)	0	53	59	59
	P_{3v} (MW)	0	0	0	0
y	P_{4p} (MW)	800	800	800	800
	P_{4v} (MW)	374	332	374	333
	P_{5p} (MW)	0	0	0	0
	P_{5v} (MW)	0	0	0	0
	l_p (€/MWh)	31.0	26.5	31.0	26.4
	l_v (€/MWh)	17.5	17.2	17.5	17.2
	D_p (MW)	1899	2353	1899	2359
	D_v (MW)	1374	1419	1374	1417

Table IV.3 Results obtained in the four cases

	a	b	c	d
J_{px} (€/MWh)	20.0	10.9	20.0	10.8
J_{vx} (€/MWh)	10.8	10.0	10.8	10.0
J_{py} (€/MWh)	23.0	18.5	23.0	18.4
J_{vy} (€/MWh)	15.0	15.0	15.0	15.0

Table IV.4 Marginal income

Finally, dual variables of maximum power limit (p^M), minimum production constraint (p^C), global market share (p^Q) and market share for every load level (p^L) are shown in table IV.5. These variables are only shown if they are positive in at least one case.

	a	b	c	d
p^M_{p1} (€/MWh)	-10.0	-20.0	-10.0	-10.8
p^M_{v1} (€/MWh)	-0.8	-10.0	-0.8	-10.0
p^M_{p2} (€/MWh)	0.0	-10.0	0.0	-0.8
p^M_{v2} (€/MWh)	0.0	0.0	0.0	0.0
p^M_{p4} (€/MWh)	-8.0	-3.5	-8.0	-3.4
p^M_{v4} (€/MWh)	0.0	0.0	0.0	0.0
p^C_3 (€/MWh)	0.0	0.0	10.0	9.2
p^Q_{e1} (€/MWh)	0.0	10.0	0.0	10.0
p^L_{ve1} (€/MWh)	0.0	9.1	0.0	0.0
p^L_{pe1} (€/MWh)	0.0	0.0	0.0	0.0

Table IV.5 Results obtained in the four cases

In all cases, it can be shown that equation (22) is valid for both companies and load levels.

The consideration of minimum market shares makes the company *x* to produce with groups that, other way, would not generate, decreasing its marginal income.

The minimum generation constraint forces group *3* to produce, even when group *2* is not at its maximum power. The interpretation of the dual variable (10 €/MWh) is that the apparent cost of group *3* is 20-10=10 €/MWh. This is the same cost as group *2*, and that is because they produce at the same time. The group *5* (with a lower generation cost than group *3*) has no changes in its production because it belongs to the other company.

V. CONCLUSIONS

A methodology that allows the computation and decomposition of marginal costs for a GENCO in a competitive framework has been presented. In order to represent the market equilibrium, a Cournot model has been used.

A number of typical operation constraints have been taken into account to analyse their influence in the marginal cost of a GENCO. The inclusion of these constraints modifies the hypothesis of the Cournot equilibrium. Despite this, the new situation gets to a Nash equilibrium, as there is a unique solution in which the unilateral modification of strategy by one agent arises to a fall in its profit.

In this new Nash equilibrium, some groups can produce although their variable costs are higher than the marginal income of the company. This is because the company perceives an apparent cost that is decreased by the incentives or the dual variables of some constraints.

GENCOs should pay attention to these economics signals. If they are ignored in the operation of the groups, the company will not achieve its objectives.

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VII. BIOGRAPHIES

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