A HYBRID APPROACH FOR MODELING ELECTRICITY PRICE SERIES IN THE MEDIUM TERM

Javier García-González Julián Barquín Pablo Dueñas
Instituto de Investigación Tecnológica (IIT)
U. Pontificia Comillas, C/ Santa Cruz de Marcenado 26, 28015 Madrid, Spain
javiergg@iit.upcomillas.es julian.barquin@iit.upcomillas.es pablo.duenas@iit.upcomillas.es

Abstract – This paper presents a hybrid approach that combines a fundamental model with an econometric analysis for modelling electricity price series in the medium-term. The fundamental model is formulated as a traditional cost-based optimization problem where the objective is to find the minimum-cost optimal dispatch of the generators, subject to system constraints. The dual variable of the demand equation provides the marginal cost of the system for each temporal period. In an ideal perfect market, the electricity price should be the marginal cost. As a consequence, marginal costs provide a useful reference with respect to which prices can be compared with. From the weekly average real prices and the simulated marginal costs, their differences can be obtained and studied under an econometric perspective. In particular, the paper studies both the logarithmic difference between market price and marginal costs, and the Lerner index. This approach has been tested in a real case, as presented in the paper.

Keywords: Price series modelling, fundamental model, power generation, electricity markets, linear programming

1 INTRODUCTION

The deregulation of power systems and the creation of spot electricity markets have given rise to new challenges and new concerns in this industrial sector. In particular, modelling and forecasting electricity prices have become a key matter, not only for market participants but also for the regulators, as part of their market monitoring duties. This fact has triggered a significant research effort, and due to the special characteristics of electricity prices, many different techniques have been proposed in the literature to study this problem. Basically, it is possible to identify two main approaches: fundamental models and quantitative models [1].

On the one hand, fundamental models can be viewed as the natural evolution of traditional cost-based engineering models where the underlying characteristics of the production assets and the demand are represented in detail. In [2] there is a complete review of these models, and it classifies them as optimization, equilibrium, and simulation models. In the medium term, the current trend is to formulate the Nash equilibrium problem to model the strategic behaviour of market participants. The major drawback of this approach is that it requires a fine-tuning of the input-parameters (such as the conjectural variation) in order to obtain a solution in accordance with the observed market results.

On the other hand, quantitative methods are based on the application of advanced statistical techniques to the available historical data. This approach is very common in the financial sector, where the dynamics of the time series are driven by stochastic processes. In [3] electricity prices models based on time series are reviewed in detail. First of all, authors discriminate between stationary and non-stationary models. Although electricity prices exhibit a well-known non-stationary component due to its multiple seasonality, several authors have proposed different stationary models for electricity prices series. For example, Dynamic Regression Models [4], Linear Transfer Function Models [4] or ARIMA models [5]. Another alternative for modelling electricity prices is to apply non-stationary models. Econometric and Financial world have given rise the most of the non-stationary electricity prices models: Mean-Reversion models [6], GARCH models [7], two factor models [8], or jump diffusion models [9]. Besides this, different authors have proved that neural networks can properly be used for modelling the evolution of the electricity prices series [10]. However the switching nature of spot prices requires to apply switching models. In this group the most important econometric model is proposed in [11] and [12] where price series is modelled through a Markovian switching process among autoregressive regimes, adapting to occasional discrete shifts in the level, variance and autoregressive dynamics of the series. In [3] a novel approach is proposed for modelling and forecasting electricity prices by the Input/Output Hidden Markov Model (IOHMM) originally proposed in [13]. The main weakness of quantitative models is that the historical data might not be rich enough to explain the future, especially in the midterm where the electric industry is subject to continuous regulatory changes.

This paper presents a hybrid approach that combines a fundamental model with an econometric analysis based on an empirical study of historical data, for modelling electricity price series in the medium-term. Let t denote the index of weekly periods, let p_t be the average system marginal price observed in the market during week t, and let λ_t be the average estimated marginal cost. In an ideal perfect market, economy theory demonstrates that prices should be equal to the marginal costs, i.e. $p_t = \lambda_t$. Therefore, the analysis of the difference between both values would reveal very useful information about market behaviour. In this paper we have analyzed the mark-up under two possible definitions: the logarithmic difference that measures the relative difference between both values (1), and the Lerner index (2), which is a well-known parameter used in economy theory to quantify the degree of competition in a market:

$$M_t = \log(p_t) - \log(\lambda_t) \quad (1)$$

$$L_t = (p_t - \lambda_t)/p_t \quad (2)$$

Market prices are public information that can be accessed easily. In our case, we have used the hourly data of the Spanish day-ahead spot market. However, the estimation of the marginal cost is not a trivial issue, as in any other business, electric companies do not reveal their private information (fuel costs, units' technical limitations, etc.). For that reason, in order to obtain the time series of the marginal cost it is necessary to make some assumptions. For example, in [12], the authors estimate the short-term marginal costs by building the hourly curves of the variable generation cost of the system, assuming that they know the heat rate of all the plants, their operating and maintenance costs (O&M), and assuming that the cost of the fossil-fuels are those negotiated daily in the international markets. In this paper we propose to build the estimation of the marginal cost by implementing an optimization model, where the dual variable of the supply-demand balance constraint provides directly its accurate value. This is quite relevant for generation systems where hydro generation can replace expensive thermal units during peak-hours. Moreover, in order to obtain price forecasts, it is not possible to know in advance the hourly demand that is going to be covered just by thermal units, and therefore, this kind of optimization model that considers simultaneously the whole generation system becomes necessary.

This paper is organized as follows. Section 2 presents a detailed mathematical formulation of the fundamental model. Section 3 presents an empirical study to estimate the marginal cost in the Spanish system in the period 2002-2005. Then, section 4 presents the econometric models developed to study the resulting time series, and the forecasting procedure is described in section 5. Then, concluding remarks are drawn in section 6.

2 FUNDAMENTAL MODEL

2.1 Representing time

The fundamental model is formulated as a traditional cost-based optimization problem where the objective is to find the scheduling of all the thermal and hydro generators of the system that satisfies the system demand, and that minimizes the operational cost in the medium term. Ideally, this model should embrace all the details relating to electricity system operation over time. This goal is not realistic, however, due to the enormous complexity of thermal and hydroelectric generation facilities. For this reason, instead of working with a total of 8760 hours per year, monotonic load curves are used to represent the different load blocks of each weekly period. Due essentially to the fact that electric power is not storable in economically significant quantities, the inclusion of information on peak demand and the ratio between each day's peak and off-peak values is of vital importance to capture the effect of the demand profile on the resulting marginal costs. The reason is that thermal sets cannot be started up and shut down indiscriminately; rather their operation is usually cyclical, normally on a week-by-week basis. Consequently, when planning for yearly demand coverage, it should be borne in mind that generating plant flexibility, and hence its ability to accommodate intraday differences in consumption, is limited. To prevent load block representation from overly distorting the actual nature of the generation-demand balance, then, a time-of-day distinction is generally drawn between weekdays and holidays; in addition, the curve may be divided into more intervals (peak, off-peak, etc.). In this paper, we have studied the Spanish case, where it is convenient to distinguish among weekdays, Saturdays, and non-working days. Therefore, the load blocks considered in this study are defined as weekly periods t, sub-periods s to distinguish among weekdays, Saturdays, and non-working days, and load blocks b to distinguish among super-peak, peak, plateau and off-peak hours.

2.2 Objective function

The objective function (3) consists of minimizing the total operational cost of the system c. This cost is made up of the fuel consumption cost c_f, the operation and maintenance cost c_{om}, the carbon emission cost c_{CO2}, and the non-served energy penalization c_{NSE}.

$$\text{Min} \quad c = c_f + c_{om} + c_{CO_2} + c_{NSE} \quad (3)$$

The system fuel cost can be expressed as shown in (4).

$$c_f = \sum_{t\in T}\sum_{s\in S}\sum_{j\in J}\left(u_{jts}\cdot f_{jt}\left(\beta_j\cdot\sum_{b\in B}a_{tsb} + \alpha_j\sum_{b\in B}\left(a_{tsb}\cdot\frac{q_{jtsb}}{k_j}\right)\right)\right)$$

(4)

where each thermal unit j is characterized by its own fuel cost f_{jt} [€/Th] that can vary trough the year. When the thermal unit is committed (u_{jts} = 1), its fuel consumption depends on the fixed term β_j and the incremental term α_j of its input-output curve. The duration of each load block is taken into account to transform the average power into generated energy. The factor k_j measures the relation between the gross power and the net power, which are different due to the internal load of the plants.

The system operation and maintenance cost of can be computed as shown in (5),

$$c_{om} = \sum_{t\in T}\sum_{s\in S}\sum_{j\in J}\left(om_j \cdot \sum_{b\in B}\left(a_{tsb} \cdot q_{jtsb} / k_j \right)\right) \quad (5)$$

where it is assumed that the variable operation and maintenance cost om_j of each thermal unit is constant, and also, that this cost is expressed in terms of the gross energy generated.

Regarding the cost of carbon dioxide emissions, equation (6) takes into account the price of CO_2, denoted by p_p^{CO2} [€/ton], and the emission rate er_j [ton/MWh] of each thermal unit.

$$c_{CO_2} = \sum_{t\in T}\sum_{s\in S}\sum_{j\in J}\left(p_t^{CO_2} \cdot er_j \cdot \sum_{b\in B}\left(a_{tsb} \cdot q_{jtsb} / k_j \right)\right) \quad (6)$$

Finally, the cost of non-served energy is computed in (7), where the per-unit cost of non-served energy is assumed to be input-data.

$$c_{NSE} = c^{NSE} \cdot \sum_{t\in T}\sum_{s\in S}\sum_{b\in B} NSP_{tsb} \cdot a_{tsb} \quad (7)$$

2.3 Constraints

The production of the generators must respect the maximum available capacity. For thermal units, this upper bound is formulated in (8), where the commitment decisions u_{jts} are included to ensure a null output power when the unit is not committed. Note that the maximum power depends on the temporal index in order to reflect the reduction of the available capacity due to programmed maintenance, or historic unit's outage when calibrating the model with historical data:

$$q_{jtsb} \le u_{jts} \cdot \overline{q}_{jt} \quad (8)$$

Hydro units are modelled by an equivalent energy representation. Historic data can provide the maximum output power. This value can vary through the year as there is a physical dependence between the maximum power and the net-head of the reservoirs, which is also related to the available hydro energy. Thus, the maximum hydro generation is stated in (9).

$$q_{htsb} \le \overline{q}_{ht} \quad (9)$$

The minimum stable load constraint of thermal units is stated in (10):

$$q_{jtsb} \ge u_{jts} \cdot \underline{q}_j \quad (10)$$

Run-of-river generation of hydro units can be imposed by the next constraint (11):

$$q_{htsb} \ge \underline{q}_{ht} \quad (11)$$

The system supply-demand balance equation is stated in (12), where λ_{tsb} stands for the dual variable of the constraint:

$$\sum_{j\in J} q_{jtsb} + \sum_{h\in H} q_{htsb} + W_{tsb} + CHP_{tsb} + NSP_{tsb} = D_{tsb} : \lambda_{tsb} \quad (12)$$

where D_{tsb} is the system demand, W_{tsb} is the wind power generation, CHP_{tsb} is the cogeneration and NSP_{tsb} is the non-served power. These terms are different at each period t, each type of day s and each load level b.

The dual variable of the demand balance equation is the system marginal cost λ_{tsb}, which is the cost of providing and additional MW in such load block.

As the aim of this model is just to obtain a reasonable estimation of the marginal cost, a very simplified representation of the hydroelectric system has been implemented in the model. From historical data it is possible to compute the available hydraulic energy for each month m. The model would optimize the allocation of this energy throughout each month, taking into account that some weeks may belong partially to two different moths (13).

$$\sum_{t\in T}\sum_{s\in S}\sum_{b\in B} b_{mt} \cdot q_{htsb} \cdot a_{tsb} \le Q_{hm} \quad (13)$$

where $b_{mt} \in [0,1]$ measures the proportion of the weekly period t belonging to the month m. Q_{hm} is the available hydraulic energy of the hydro unit h, at the month m.

Finally, as a consequence of the Spanish regulation of capacity payments, constraint (14) forces a minimum yearly production expressed in terms of number of equivalent hours h_j operating at maximum output power.

$$\sum_{t\in T}\sum_{s\in S}\sum_{b\in B} q_{tsb} \cdot a_{tsb} \ge \overline{q}_{jt} \cdot h_j \quad (14)$$

3 EMPIRICAL ANALYSIS

The optimization model has been implemented in GAMS/CPLEX-10.2. To run the model, it has been necessary to introduce as input data an estimation of the technical characteristics of the units as well as of fuel costs. These data have been obtained from regulators' and market reports and other public sources. Moreover, the maximum capacity of the units has been estimated by studying the output power of each unit cleared in the Spanish day-ahead market. This way, it has been possible to estimate historical forced outages.

Marginal costs are computed by solving the optimization problem relaxing the binary variables, which may take values in the interval [0, 1], as suggested in [14]. Figure 1 shows the solution of the optimization problem and its comparison to the historic price series.

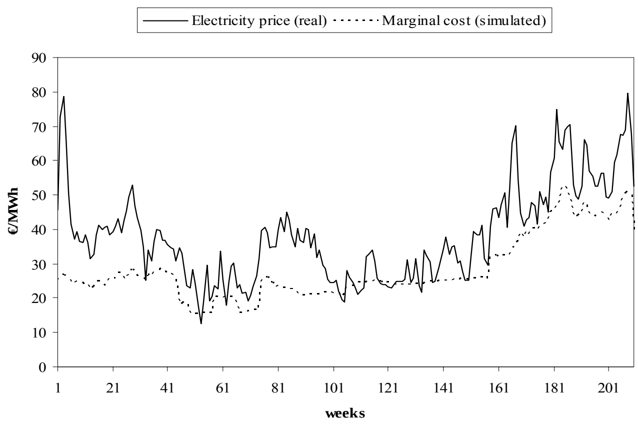

Figure 1: Real electricity price series and simulated marginal cost series from 01/01/2005 to 31/12/2005

Dual variables λ_{tsb} are obtained for each load block. Therefore, weekly average marginal costs can be computed *ex post* as follows:

$$\lambda_t = \frac{\sum_{s\in S}\sum_{b\in B}\left[\lambda_{tsb}\cdot a_{tsb}\cdot D_{tsb}\right]}{\sum_{s\in S}\sum_{b\in B}\left[a_{tsb}\cdot D_{tsb}\right]} \quad (15)$$

Weekly average market prices are obtained in a similar way from hourly prices and system demand.

4 ECONOMETRIC MODEL

4.1 Autoregressive model

From the estimated weekly marginal costs and the historical market prices, time series M_t and L_t can be computed, (1) and (2). From the autocorrelation analysis, it can be concluded that both series follow a three-order autoregressive model (16). Table 1 shows the estimated parameters $\phi_i, i = 1,..,3$ for each series:

$$\begin{aligned} M_t &= \phi_1\cdot M_{t-1} + \phi_2\cdot M_{t-2} + \phi_3\cdot M_{t-3} + a_t \\ L_t &= \phi_1\cdot L_{t-1} + \phi_2\cdot L_{t-2} + \phi_3\cdot L_{t-3} + a_t \end{aligned} \quad (16)$$

where a_t is a white noise process, which follows a normal distribution with zero mean and constant variance.

M LOGARITHMIC DIFFERENCE		*L* LERNER INDEX	
Parameters	Estimate	Parameters	Estimate
ϕ_1	0.8775	ϕ_1	0.8690
ϕ_2	-0.2098	ϕ_2	-0.2133
ϕ_3	0.2723	ϕ_3	0.2868

Table 1: Estimation of the parameters (AR model)

The estimated values of these parameters have been obtained from the *arx* function, which is included in the System Identification Toolbox of Matlab.

4.2 Explanatory variables

In order to improve the quality of the model, we have included the dependence of both series on some potential explanatory variables (ARX model): system demand *D*, the generated hydraulic power *H*, the generated wind power *W* and the generated nuclear power *N*.

Several ARX models were tested. They included combinations of different specifications for the autoregressive and explanatory variable parts. Orders for the autoregressive part ranged from 1 to 3. Several combinations of explanatory variables were tested, rejecting the ones with a significance level above 5%. Orders for the lagged explanatory variables part ranged from 0 to 2.

The best model has an autoregressive part of order 3 and two explanatory variables. The first one is the demand D minus generated nuclear power *N*. This new variable is denoted by *DN*, (17). The second one is the increase of the generated hydraulic power between consecutive weeks denoted by *HH*, (18).

$$DN_t = D_t - N_t \quad (17)$$

$$HH_t = H_t - H_{t-1} \quad (18)$$

We are not aware of prior usage of variable *HH* in other works. Actually, we tested the introduction of generated hydraulic power *H* as explanatory variable. When one or two lags were tested, the first and the second parameter were similar, but they had a different sign. This fact motivated the inclusion of this new variable in the ARX model.

Finally, we tested also the inclusion of generated wind power, *W*, as this technology has played an important role in the Spanish system during the last years. Surprisingly, the statistical test did not support the inclusion of this variable in the ARX model. This fact could be a sign of a problem, possibly because of the lack of reliable data.

Table 2 shows the final estimation of the parameters of the ARX model.

Parameters	*M* LOGARITHMIC DIFFERENCE		*L* LERNER INDEX	
	Estimate	Sign. Level	Estimate	Sign. Level
ϕ_1	0.8148	0.000	0.7908	0.000
ϕ_2	-0.2126	0.013	-0.2007	0.018
ϕ_3	0.1768	0.008	0.1599	0.016
DN_t	$4.01\cdot10^{-6}$	0.000	$5.88\cdot10^{-6}$	0.001
HH_t	$4.31\cdot10^{-5}$	0.000	$4.07\cdot10^{-5}$	0.000

Table 2: Estimation of the parameters (ARX model)

The significance level is calculated by the statistic t^* (19), which is compared with a Student's t distribution of $T-p$ degrees of freedom, where T is the number of periods and p the autoregressive order.

$$t^* = \frac{\hat{\phi}}{\hat{\sigma}_{\hat{\phi}}} \quad (19)$$

4.3 Model validation

The model diagnostics requires a visual verification (Figure 2) and a test of the basic hypothesis made respect to the residuals, which is that the innovations follow a white noise process, i.e. they must be uncorrelated at any time lag (Figure 3) and normal distributed with zero mean and constant variance (Figure 4). For both models, the graphs are similar. As a consequence, only the graphs of the logarithmic difference are shown.

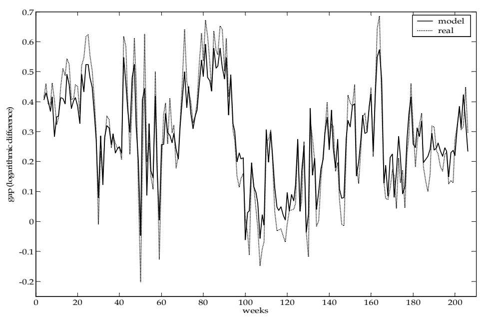

Figure 2: Visual verification of the model

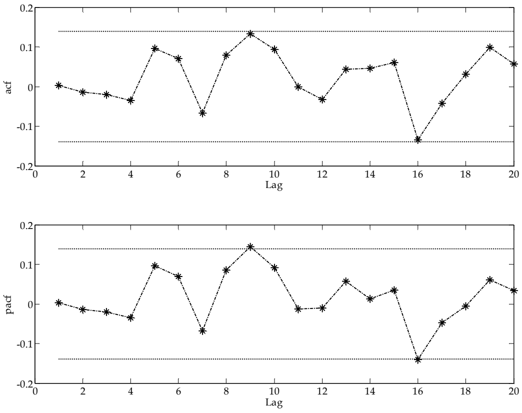

Figure 3: Autocorrelation function of the model innovations

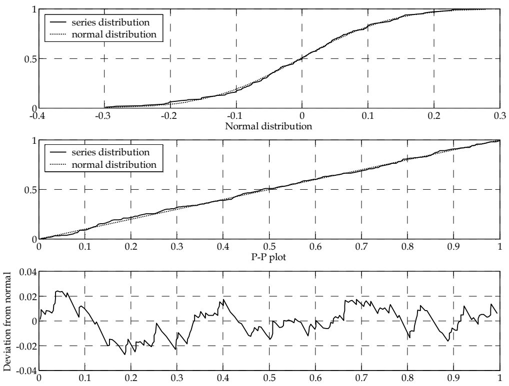

Figure 4: Normality test of the innovations

4.4 AR and ARX models discussion

Comparing the residual variances of AR and ARX models, it can be observed the reduction of the error due to the inclusion of explanatory variables, see Table 3.

	LOGARITHMIC DIFFERENCE	LERNER INDEX
	Residual variance (σ)	Residual variance (σ)
AR	0.0179	0.0113
ARX	0.0152	0.0097
Difference	-15.08%	-14.16%

Table 3: Comparison of the residual variances

Furthermore, the signs of the parameters associated to each explanatory variable in the model, are intuitively coherent with their expected influence on the mark-up. On the one hand, the positive sign of the *DN* indicates that the higher the demand, the greater the mark-up, or the greater the nuclear power, the lower the mark-up. On the other hand, the positive sign of the weekly variation of the generated hydraulic generation indicates that as more hydraulic power is generated respect to the last week, the greater the mark-up.

5 FORECASTING

In order to obtain forecast of market prices one-year in advance, it would be necessary to:

- Run the fundamental model in order to obtain an estimation of the marginal costs.
- Have an estimation of the explanatory variables (*DN* and *HH*) to compute the mark-up.

Note that both tasks require selecting the most relevant scenarios of the fundamental drivers (demand, nuclear availability, fuel costs, hydro inflows, etc.).

Once both tasks have been performed, marginal price could be forecasted by adding the mark-up computed by the ARX model, to the marginal cost obtained by the fundamental model. This can be done solving for the price in (1) or (2).

We have attempted to apply this methodology to a realistic case. To forecast year 2005 the parameters of the model have been adjusted by using data from 2002-2004. A similar model than the one presented in the previous section has been adjusted. However, as the values of fundamental drives are the real ones, in this example, the forecast error might be smaller than the one that would be obtained in a more realistic setting.

Figure 6 shows the price forecasting from 01/01/2005 to 31/12/2005. The dash line represents the real prices. The thick central line is the expected price. The thick lower and upper lines bound the region where the price is expected to lie with a 68% (1σ) of probability.

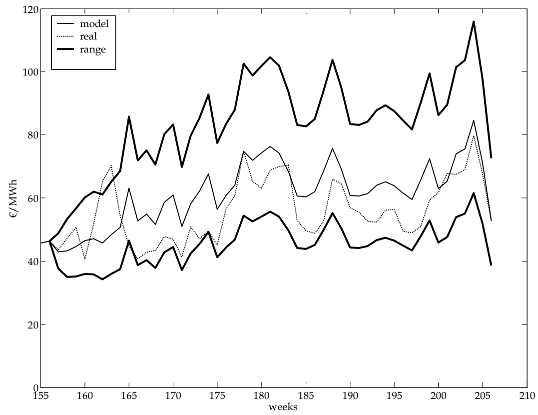

Figure 5: One-year price forecasting of 2005

6 CONCLUSIONS

This paper presents a novel approach for modelling prices in competitive electricity markets, by combining a fundamental model with an econometric analysis. The fundamental model has been implemented as an optimization problem that allows to obtain the theoretic marginal cost of the system as the dual variable of the demand balance equation. The econometric model is implemented as an AR or ARX process of the logarithmic mark-up and the Lerner index, obtained from the historical prices and estimated marginal costs. Inputs required for the calibration of both models have been obtained from public available historical data.

The econometric models provide relevant information about market behaviour. Moreover, they can be applied for forecasting purposes given some hypothesis about the expected evolution of the fundamental drivers (demand, fuel costs, hydro inflows, etc.).

7 APPENDIX

Hereafter it is presented the notation used throughout the paper:

7.1 Sets and indexes:

M, m Set and index of months

T, t Set and index of weekly periods

S, s Set and index of sub-periods (working days, Saturdays, and non-working days)

B, b Set and index of load blocks (super-peak, peak, plateau, and off-peak hours)

J, j Set and index of thermal units

H, h Set and index of hydro units

7.2 Parameters and constants:

For each of thermal unit *j*

α_j Slope of the input-output curve [Th /MWh]

β_j Fixed term in the input-output curve [Th /h]

k_j Gross to net capacity conversion factor [p.u.]

om_j Operation and maintenance costs [€/MWh]

er_j Emission rate [ton/MWh]

For each of thermal unit *j* in period *t*

$\overline{q}_{jt}$ Maximum output power [MW]

$\underline{q}_{jt}$ Minimum stable load [MW]

f_{jt} Cost of fuel in the period *t* [€/Th]

For each of hydro unit *h* in period *t*

$\overline{q}_{ht}$ Maximum output power [MW]

$\underline{q}_{ht}$ Minimum stable load [MW]

For each load block *b*, in sub-period *s*, of period *t*

a_{tsb} Duration [h]

D_{tsb} Demand [MW]

W_{tsb} Wind generation [MW]

CHP_{tsb} Cogeneration [MW]

NSP_{tsb} Non-served power [MW]

Others

Q_{hm} Available hydro power of hydro unit *h* in the month *m* [MWh]

$p_t^{CO_2}$ Price of CO_2 in period *t* [€/ton]

c^{NSE} Non-served energy cost [€/MWh]

7.3 Variables:

λ_{tsb} System marginal cost at the load block *b*, in sub-period *s*, of period *t* [€/MWh]

q_{jtsb} Power generation of thermal unit *j* at the load block *b*, in sub-period *s*, of period *t* [MW]

q_{htsb} Power generation of hydro unit *h* at the load block *b*, in sub-period *s*, of period *t* [MW]

u_{jts} Binary variable (0, 1) that indicates if the thermal unit *j* is committed in the sub-period *s*, of period *t*, or not

b_{mt} Measures the proportion of the weekly period *t* belonging to the month *m*

REFERENCES

[1] D. Bunn, "Modeling prices in competitive electricity markets," John Wiley & Sons, 2004.

[2] M. Ventosa, A. Baíllo, A. Ramos, M. Rivier, “Electricity market modeling trends”, Energy Policy, 33, 2005, pp. 897-913

[3] Mateo, A., Muñoz A., and García-González, J.. (2005). "Modeling and Forecasting Electricity Prices with Input/Output Hidden Markov Models." IEEE Transactions On Power System Vol. 20, No. 1.

[4] Nogales, F. J., Contreras, J., et al. (2002). "Forecasting Next-Day Electricity Prices Series by Time Series Models." IEEE Transactions On Power System 17(2)

[5] Contreras, J., Espinola, R., Nogales, F. J, Conejo, A J. (2003). "ARIMA Models to Predict Next-Day Electricity Prices." IEEE Transactions On Power System, Vol 18, No 3.

[6] Knittel, C. R. and Roberts, M. R. (2001). An Empirical Examination of Deregulated Electricity Prices. California, University of California, Berkeley, www.ucei.berkeley.edu.

[7] Batlle, C. and Barquín, J. (2002). Fuel prices scenario generation based in a multivariate GARCH model for risk analysis in a wholesale electricity market. 7th International conference on Probabilistic Methods Applied to Power Systems (PMAPS), Naples.

[8] Schwartz, E. and Smith, J. E. (2000). "Short-Term Variations and Long-Term Dynamics in Commodity Prices." Management Science 46(7): 893-911.

[9] Deng, S. (2000). Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes, www.ucei.berkeley.edu.

[10] Szkuta, B., R., Sanabria, L., A., et al. (1999). "Electricity Price Short-Term Forecasting Using Artificial Neural Networks." IEEE Transactions On Power System 14(3): 851-857.

[11] Hamilton, J. D. (1990). "Analysis of time series subject to change in regime." Journal of Econometrics 45: 39-70.

[12] Fabra, N. and Toro, J. (2002). Price Wars and Collusion in the Spanish Electricity Market. Madrid, Fundación Empresa Pública.

[13] Bengio, J. and Frasconi, P. (1996). "Input-Output HMM's for Sequence Processing." IEEE Transactions on Neural Networks 7(5).

[14] Vázquez, C. (2003) "Clearing market models in electricity markets”, in Spanish" (in Spanish). Ph. D. Thesis, Pontificia Comillas University, Madrid, Spain, September 2003.