APPLICATION OF DISCONTINUOUS DEFORMATION ANALYSIS TO NUMERICAL SIMULATION OF GRANULAR MATERIAL

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Key Words: Granular Material, Discontinuous Deformation Analysis, Numerical Simulation, Gravity Table Separator, Compost, Vibrating Feeder.

ABSTRACT

This paper presents a numerical model for the simulation of granular material movement. Granular material can be defined as the material formed by multiple solid particles and it represents the most common form of raw material in industry world-wide. The proposed computer model is based on Discontinuous Deformation Analysis, a numerical method recently adapted for simulation of rigid particles in motion. Gravity, friction, cohesion, impacts between particles and impacts with the boundaries are considered. DDA is based on a system of discontinuous elements where displacements of each block are the unknowns and are obtained solving the equilibrium equations. These equations are formulated through minimizing the total potential energy of the system, in a similar fashion to FEM. The motion of the particles is induced by the vibration of the deck on which they rest and boundary conditions are set to model a vibrating feeder for granular material transport. The model is compared to a simple analytical model used by industries, showing that qualitative results agree. The model permits to calculate the optimum machine parameters for efficient operation. Numerical results were applied to optimize the design of a gravity-table machine for compost of which already a first prototype has been constructed.

1.- INTRODUCTION

Granular material can be defined as the material formed by multiple solid particles, whatever the size of the particles. It represents the most common form of raw material in industry worldwide. In weight, it is estimated that at least 75% of the raw material and 50% of the products in the chemical industry (Nedderman 1992) are handled as granular material. Chemical, pharmaceutical, food, mining, metallurgy, construction and many other industries use it continuously in its applications and processes. Sugar, salt, flour, cement, sand and pills are worth examples of granular material.

Although its simple appearance, granular material presents a wide range of behaviors that are still far from being full understood and explored. This form

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of material is considered by some experts as a fourth state of matter; it can behave as a solid (it undergoes deformation under loads); as a liquid (it flows and is poured); as a gas (it has compressibility and is formed by particles without bonds);or more dramatically in its unique way. In spite of these fascinating properties, scientists and researchers have paid little attention to it.

In opposition to fluid dynamics, granular dynamics was put partially aside until computer revolution in the eighties allowed a more successful approach to the topic. Just one decade ago, industrial structures related to granular mechanics as silos, stockpiles, feeders or conveyors were based on simple and low performance experimental results with little theoretical basis. Nowadays, technology and computer-based simulations permit a more efficient approach towards structure optimization based on more precise and rigorous results.

Among machines for material handling (conveyors, tables, bins, elevators, etc), this paper focuses on vibrating feeders. These transports move material inducing movement by a vibrating deck in contact with the particles.

A gravity table is a machine for separation of granular material in dense and less dense (see Fig. 1) by two physical actions: vibrating transport and fluidization. This paper is based on the research conducted for optimization of a gravity table separator, focusing on the vibrating feeder that drives the discharge of the dense phase.

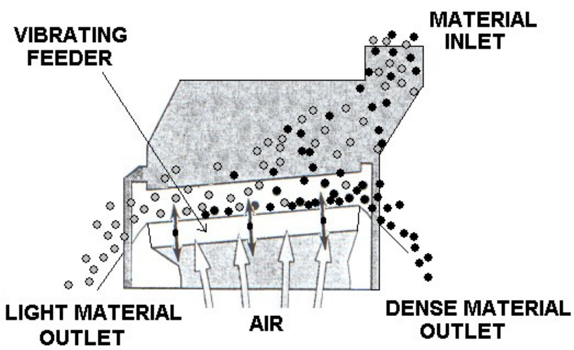


Fig. 1. Schematic operating gravity table separator.

2.- DISCONTINUOUS DEFORMATION ANALYSIS

A new numerical method was presented in the mid eighties (Shi et al. 1984) called Discontinuous Deformation Analysis (DDA). In some aspects, this method is similar to the finite element method (FEM) and in other aspects to the discrete element method (DEM). It was first developed for applications in geomechanics for the study of landslides and rocks motions (Fig. 2a and 2b) (Shi 1993). Today, it has been successfully applied to other engineering fields as for example, bulk material discharge (Fig. 2c) (Fernández 1997).

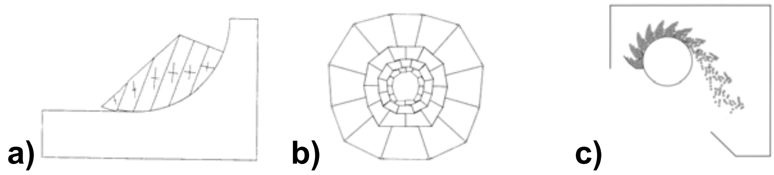


Fig. 2. DDA examples: a) landslides; b) tunnel; c) bulk material discharge.

DDA is based on a system of discontinuous elements where displacements of each block are the unknowns that are obtained solving the equilibrium equations. These equations are formulated through minimizing the total potential energy of the system (as FEM). The formulation is fundamentally discontinuous in the sense of the elements (as DEM). Advantages of this method are (Thomas 1996): dynamic equilibrium is always reached; penetration of bodies and traction on surfaces are not permitted at each time step; and solution stability is reached without introducing artificial damping.

This method was further developed (Ke et al. 1995) to a formulation specially adapted to two dimensional rigid disks and rigid moving boundaries. The model presented in this paper is based on this last formulation of the DDA method.

Detailed formulation of the method can be found in (Ke et al. 1995).

In the reference coordinate system of a disk, u_1 and u_2 are the displacements of point *i*, identified by its coordinates (x,y). Each disk has three degrees of freedom: two translations (U_1,U_2) and one rotation U_3. Relation between point displacements and disk movement is given by the first order approximation (1).

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}^i_{(x,y)} = \begin{pmatrix} 1 & 0 & (y_0-y) \\ 0 & 1 & (x-x_0) \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}^i = \begin{pmatrix} 1 & 0 & (y_0-y) \\ 0 & 1 & (x-x_0) \end{pmatrix} [D_i] \qquad (1)$$

If there are *n* disks, a system of *n* equations is formed in terms of the $[D_i]$ displacements submatrices with the correspondent stiffness $[K_{ij}]$ and forces $[F_i]$ submatrices:

$$\begin{pmatrix} [K_{11}] & [K_{12}] & [K_{13}] & \dots & [K_{1n}] \\ [K_{21}] & [K_{22}] & [K_{23}] & \dots & [K_{2n}] \\ [K_{31}] & [K_{32}] & [K_{33}] & \dots & [K_{3n}] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ [K_{n1}] & [K_{n2}] & [K_{n3}] & \dots & [K_{nn}] \end{pmatrix} \begin{pmatrix} [D_1] \\ [D_2] \\ [D_3] \\ \vdots \\ [D_n] \end{pmatrix} = \begin{pmatrix} [F_1] \\ [F_2] \\ [F_3] \\ \vdots \\ [F_n] \end{pmatrix} \qquad (2)$$

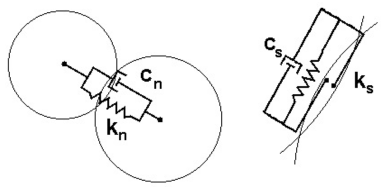
$$[K_{ij}] = \sum_{l=1}^{L} [K_{ij}]^l \qquad (3)$$

$$[F_i] = \sum_{l=1}^{L} [F_i]^l ;\text{with } l \text{ enegy sources} \qquad (4)$$

Each row of equation (2) is obtained minimizing the potential energy *E* (5) produced by all the forces acting (eq. (3) and (4)). Submatrices $[K_{ii}]$ depends on the material properties of disk i and $[K_{ij}]$, where $(i \neq j)$, is defined by contact between disks *i* and *j*.

$$\frac{\partial E}{\partial U_j^i} = 0,\ \ j = 1,2,3\ \text{ for disk }\ i \tag{5}$$

Surface contact is modeled by means of two visco-elastic elements (Fig.3) which are added to the global system matrix (2) in the time step when contact is detected (penalty method). This methodology fulfills constraints of no tension between blocks and no penetration of one block into another.



(a) (b)

Fig 3. Contact elements: a) normal and b) tangential.

Contact and interaction between multiple bodies introduces high non-linearity into the problem. To control this non-linearity, at each time step the algorithm checks contact states and constrains. In case constrains are not completely fulfilled, calculation process is repeated changing the state of contacts from open to close or vice versa and reducing the time increment until convergence is reached.

Disadvantages of DDA method are time and memory requirements. Comparing extreme cases to FEM, DDA is five times slower and memory needed is four times larger.

3.- APPLICATION TO GRANULAR MATERIAL TRANSPORT IN VIBRATING FEEDERS

The model was applied to the design of a gravity table separator using the simulation results. Some machine parameters were externally imposed by mechanical design criteria and values were directly introduced into the model. Material properties were determined by consulting available literature and studies of the interesting mechanical design parameters were conducted. In order to better improve the material model, particles have been arranged forming different clusters (Fig. 4a).

The model simulates a 2D rectangular compartment that contains a plane deck inclined an angle α with the horizontal direction (Fig. 4b). The deck is the feeder that is excited with a sinusoidal vibration in the β angle direction whereas a sufficient quantity of particles with similar properties to the real material is placed on the deck. Deck starts to move and particles react. The objective is to maximize the horizontal discharge velocity. Angle □ of the feeder must be small but not zero, because the gravity separation phenomenon needs a slight inclination in order to be efficient and move forward only the dense phase of the material.

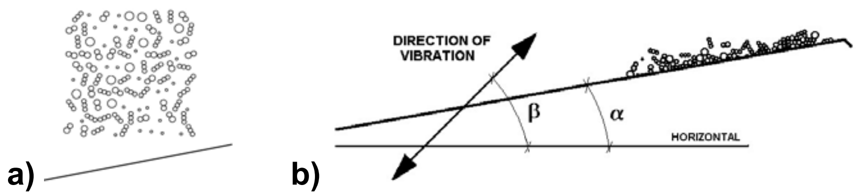


Fig. 4. a); Particle clusters; b) Geometrical parameters of the model.

Initial conditions were set to zero velocity for both the disks and the boundaries.

In the simulations (Fig. 5), discharge velocity is computed deriving numerically the horizontal position of the center of gravity respect to time. The graph of the velocity versus time (Fig. 6) shows a short unsteady period followed by a longer quasi-steady period, consequently, simulations were carried out until useful information about the steady regime could be drawn.

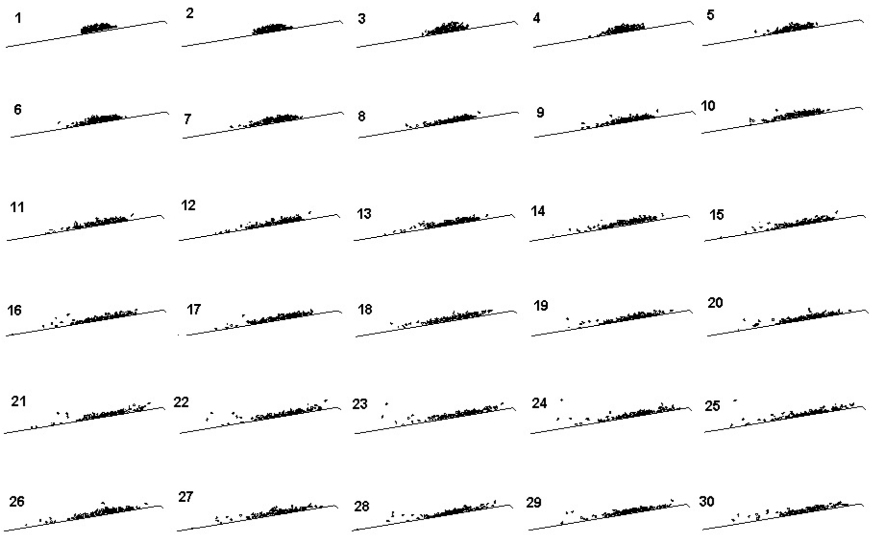


Fig. 5. Sequence of instances for α =10º, β =22º and Γ =6.18 (eq. (6)).

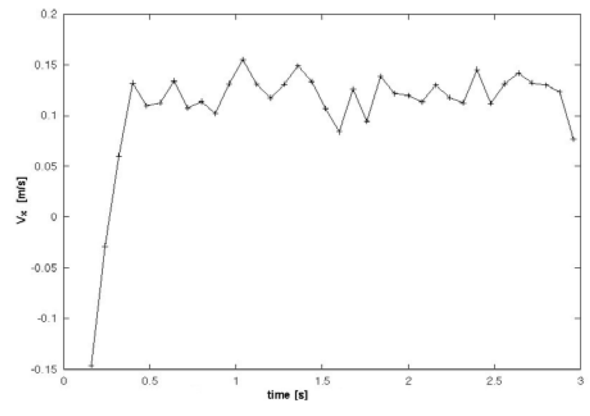


Fig. 6. Horizontal velocity of the center of gravity for α =10º, β =22º and Γ =6.18 (eq. (6)).

The model permits to analyze machine parameters and material properties effects through parametrical and sensitivity studies. Here is a brief presentation of the different parameters that the model requires and therefore they can be studied. The material used in the simulations represents the dense phase of compost. Compost is the fermented organic fraction of solid urban waste.

See Table 1 for material properties. Material is formed of 175 disks forming clusters. The density represents the mean density of glass, plastic and light metals. Values for friction and cohesion of particles were found in literature. The damping of the system results in an overall restitution coefficient near 0.8.

Table 1. Material properties	
Shape and size	Diameter between 5 and 10 mm
Density	2500 kg/m^3
Particle friction coefficient	0.84 (•)
Particle cohesion	700 N/m^2 (•)
Restitution coefficient	Near 0.8

See Table 2 for machine operating parameters. Angle α for deck inclination respect horizontal, vibration amplitude and vibration frequency were imposed by mechanical design criterion. The resulting non-dimensional acceleration number Γ (6) is the ratio of vibrating acceleration to gravity acceleration g. *A*ngle β of vibration respect horizontal is a variable parameter during operation. The optimum value for β is discussed in the next section. The friction of the deck represents that of a metal but increased due to the holes of the screen on the deck surface. Adhesion of the deck were set to a low value.

Table 2. Machine Operating Parameters	
Angle α	8°
Vibration amplitude A	6 mm
Vibration frequency f	16 Hz
Γ	6.18
Optimum angle β	30º
Deck friction coefficient	0.65

(•) Nedderman 1992

Deck adhesion	60 N/m^2

$$\Gamma = \frac{4A(\pi f)^2}{g} \qquad (6)$$

4.- COMPUTER SIMULATION RESULTS

4.1 Validation

In order to validate the model and its results, a simple analytical model (Rachner 1964) has been used which computes the trajectory of one frictionless particle excited by a vibrating deck under gravity effect. This analytical model is conservative and non linear, although it only uses the parameters amplitude, frequency and β angle. Therefore it does not take into account critical granular phenomena like multiple particles or friction. Due to these characteristics, the analytical model is not acceptable for quantitative validation. In spite of this, the analytical model is applied for engineering applications by many industries, and thus it can be used for qualitative validation of the proposed model.

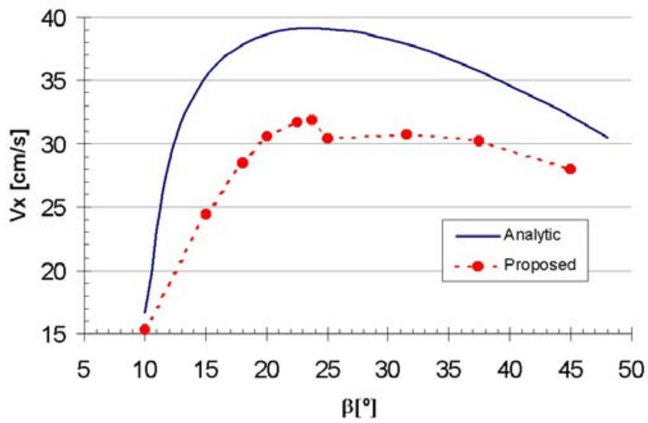


Fig 7. Results comparison as a function of angle β for α =0º, A =6 mm and f =16 Hz.

It has been compared the horizontal velocity for a similar configuration (α =0º) for both models. Results show (Fig. 7) that both models behave in a similar way, being the maximum discrepancy 38% and the mean discrepancy 16%. The optimum β angle for the proposed model is 23.75º and for the analytical model is 24º, with a slight difference of 1%. It is necessary to point out that each engineering department modifies output velocities of the analytical model multiplying by near-one coefficients related to their experience-based knowledge. These coefficients were not available for this work.

4.2 Applied Results

Simulating the dense phase of compost and introducing the machine parameters imposed by mechanical design criteria (α=8º), the model has permitted to calculate the optimum β angle of vibration. The optimum β angle, which gives the maximum output velocity, is located near 30º (Fig. 8). The increase in 8º of inclination by angle α has decreased the discharge velocity in 33% comparing to Fig. 7.

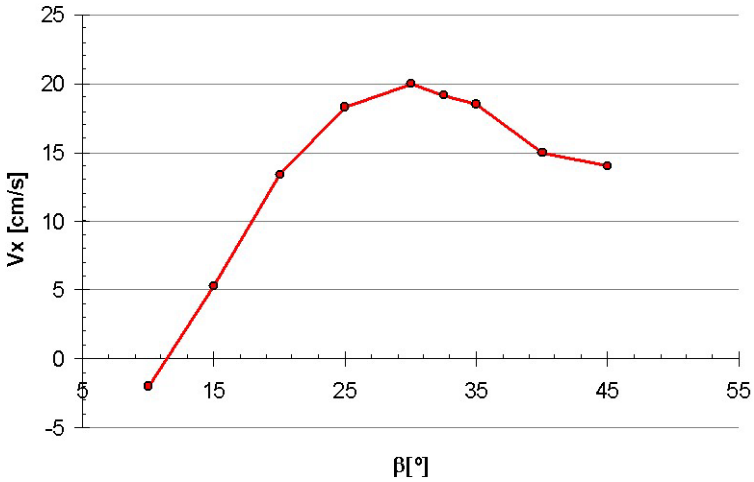


Fig 8. Discharge velocity results as a function of the angle β of vibration for α=8º and Γ=6.18.

5.- CONCLUSIONS

DDA method has been successfully applied to granular material transport simulations. Results of the model compare qualitatively well to a simple analytical model. Applied to a gravity table separator for compost, the model has allowed determining the optimum angle of vibration. Using the model, mechanical specifications of the machine has been proved to be correct and forecasted discharge velocities for dense phase are better than those of machines currently in use.

This research has permitted the construction of a first prototype (Fig. 9) that will be followed by a commercial machine.

Fig. 9. Manufactured prototype for testing

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